Digital Phase Modulation and Demodulation

Zartash Afzal Uzmi, *Lahore University of Management Sciences, Pakistan*

INTRODUCTION

In computer and communication networks, where different layers communicate at the peer level, a study of physical layer is necessary to understand the concept of information communication at the most fundamental level and how information is physically communicated from one point to another. Communication at the physical layer is the most fundamental way of providing connectivity to numerous hosts in a computer network such as the Internet. The physical layer provides connectivity between two directly connected nodes whether they are connected through a wire as in the IEEE 802.3 standard or through a wireless link as in the IEEE 802.11 standards.

In a computer network such as the Internet, end systems communicate with each other through a network that is largely a collection of communication links connected through packet switches. In almost all cases, the communication between end systems will be in the form of packets. At times, a circuit switch is used to provide an interface to end users, as in the case of users communicating through dialup connections. In a layered architecture, the network layer and the layers above it provide end-to-end connectivity between the end systems. The data-link layer, however, provides connectivity between two directly connected nodes, where a node can be an end system or an intermediate switch. The data-link layer is responsible for transporting physical frames between two directly connected or adjacent nodes. A frame is nothing more than a sequence of bits; for transporting a frame between two adjacent nodes, the data-link layer uses the services of a physical layer that is responsible for physically transporting individual bits.

In this chapter, we will consider communication of digital data only at the physical layer. This can be accomplished by various methods, one of which is *digital phase modulation* (DPM). Sometimes, digital phase modulation is used synonymously with *phase shift keying* (PSK), which is only a special case of DPM. Our focus will be on the generation and detection of digital phase modulated signals and their various properties. In the next section, we provide background for understanding communications at the physical layer, eventually leading to a definition of digital phase modulation.

BACKGROUND

At the physical layer, the communication of digital data between two directly connected nodes is carried out in one of the two fundamental ways. The first way is by using baseband communication whereby the digital data are transmitted without altering the frequency spectrum of the digital message signal or the digital baseband signal. The baseband digital communication (also known as *baseband pulse transmission*) is, therefore, a direct transportation of a digital baseband signal that is generated by directly representing digital symbols in one of the various electrical pulse shapes. Baseband digital transmission is carried over baseband channels, which are usually low-pass in nature. The second way of transporting digital data is by means of passband communication whereby the digital data to be transported are used to generate a passband digital signal that is then transmitted to the receiver.

A passband digital signal is generated by using the digital data symbols to alter some characteristics of a carrier, which is usually sinusoidal. The characteristics of a sinusoidal carrier that may be modified by the digital data include the amplitude, phase, and frequency, resulting in digital amplitude, digital phase, and digital frequency modulation, respectively. Thus, we define *modulation* as the process in which a message signal alters the properties of a carrier signal (Wikipedia 2007, "Modulation"). As a result of modulation, the frequency spectrum of the modulated signal is usually quite different from that of the baseband signal or message signal. The spectrum of the modulated signal may be obtained from the spectrum of the baseband signal in a way that depends on the type of modulation and the carrier frequency.

The message signal or the modulating signal can either be analog or digital. However, in computer networks, the message signal is almost always digital: Symbols are selected from a discrete set and sent one after the other every symbol period (or signaling period). The modulation in which the modulating signal is digital is called *digital modulation*. Because a digital signal is obtained by choosing one symbol from a discrete set, a simple way of performing digital modulation is to switch the amplitude, frequency, or phase of the carrier based on the symbol selected from that discrete set. In this form, digital modulation is also termed as *keying*. In *amplitude shift keying*, for example, the amplitude of the carrier wave is switched every symbol period from one value to another based on a symbol selected from the discrete set, which includes all possible digital symbol values.

We may now define a special form of digital phase modulation called *phase shift keying* (PSK). In PSK, the phase of a carrier—usually sinusoidal—is shifted at the start of each symbol interval directly in accordance with the digital data that need to be communicated from transmitter to the receiver in that interval (Wikipedia 2007, "Phase"). If the digital data are binary, there are only two possible symbol values (typically, 0 and 1), and the simplest way to achieve PSK is to transmit one of the two possible sinusoidal waves phase shifted by 180 degrees with respect to each other when one of the two possible symbols is to be sent. In PSK, the information to be communicated is exclusively encoded in the phase of the carrier at the beginning of every symbol interval. The envelope remains constant in a PSK signal, while the frequency also remains constant throughout a given symbol. However, an abrupt phase transition happens at symbol boundaries, causing frequency impulses.

Other modulation methods exist, in addition to PSK, in which the information can be exclusively sent in the phase of the carrier. All such digital modulation methods in which the phase of the carrier contains the information to be transferred are classified as *digital phase modulation*. By this definition, the amplitude of a DPM signal should remain constant. Because phase and frequency are related to each other, a generic digital phase modulation may also be viewed as a digital frequency modulation scheme. In particular, *continuous phase modulation* (CPM) is a nonlinear modulation scheme that falls under phase as well as frequency modulation.

Digital phase modulation can either be linear or nonlinear. Linear DPM can further be *without* memory (also known as *coherent PSK* with *binary phase shift keying, quadriphase shift keying*, and *offset quadriphase shift keying* as examples) or *with* memory (such as differential PSK). In nonlinear digital phase modulation with memory, the modulated signal is constrained to maintain a constant envelope and continuous phase, avoiding abrupt changes in phase. CPM is an example of nonlinear digital phase modulation with memory. A special case of CPM is *continuous phase frequency shift keying* (CPFSK). A famous modulation scheme called *minimum shift keying* is a further special case of CPFSK.

Coherent PSK is the simplest form of digital phase modulation and is used whenever the receiver is able to locally generate the carrier with correct frequency and phase; otherwise, the differential form is preferred. If the coherent form is used, then the demodulator must track the carrier by using a carrier recovery circuit. Any errors in tracking lead to performance degradation of the overall communication, resulting in higher bit error rates. Demodulation of CPM signals is usually accomplished by using coherent demodulators followed by sequence detectors.

A description of DPM requires understanding the terminology used in a digital communication system. We therefore first consider a baseband communication system to introduce the terminology before we set out to describe the DPM techniques in more detail.

DIGITAL COMMUNICATION SYSTEMS

In a digital communication system, the information to be sent from transmitter to receiver is always encoded in the form of digital data. If the information to be sent is already in the digital format, then the sender is not required to take any extra effort when using the digital communication system. If, however, the information to be sent is analog, such as the voice in a *voice over Internet protocol* (or VoIP) call between two end users, it is the responsibility of the sender to convert the analog information (voice) into digital format by employing analog-to-digital conversion. Obviously, the receiver will need to reverse the process by constructing an analog signal from the received digital signal.

Baseband Signal Representation

To become familiar with the terminology of digital communication systems, we start by defining a baseband digital signal as follows:

$$x(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nD)$$

Usually, this signal will be available at the output of an analog-to-digital converter. This signal is also known as a digital *pulse amplitude modulation* (PAM) signal and is transmitted as is in a baseband digital system. However, in passband digital systems such as PSK, the signal $x(t)$ is fed to a modulator before subsequent transmission. In the above expression, $x(t)$ is the baseband digital signal, $p(t)$ is the pulse shape, and a_n is the nth digital symbol that needs to be sent from the transmitter to the receiver. For a binary digital system, the symbol a_n represents one single bit and will take one of the two possible values—that is, a_n will belong to a set whose cardinality is 2. In many cases, a_n will be a symbol representing more than a single bit; in such cases, a_n will belong to a set with higher cardinality. Note that D is the duration of one symbol in the above expression, and its inverse is called the *symbol rate*. For binary digital systems, each symbol will represent a single bit, and the symbol rate will be equal to the *bit rate*. However, if a_n belongs to a set with cardinality 4, then each symbol a_n will represent two binary digits (bits), and the bit rate will be twice the symbol rate. In general, if a_n belongs to a set with cardinality M, then the bit rate is $\log_2 M$ times the symbol rate. We usually use T_b for bit duration, where T_b is related to D for an M-ary digital signal as $D = T_b \log_2 M$. Obviously, for binary signals, $M = 2$ and $D = T_b$.

Line Codes

It is important to understand the baseband digital formats (also referred to as *line codes*) because the passband digital signals (including digital phase modulation or PSK) are generated by modulating a carrier according to the baseband digital signal. Line codes or baseband digital formats are defined by selecting the parameters a_n and $p(t)$ in the expression for a baseband digital signal. Spectral properties of line codes depend on the pulse shape $p(t)$, the symbol values a_n, and the symbol rate $1/D$, and they play an important role in signal transmission. In this chapter, we only describe line codes that are of relevance to the material presented here. For a complete reference of line codes, see Barry, Lee, and Messerschmitt (2003).

The *non-return-to-zero* (NRZ) or *unipolar NRZ* or *nonpolar NRZ* format is characterized by $a_n \in \{0, 1\}$ and $p(t) = A\Pi(t/D)$ or any time-shifted version, where $\Pi(t)$ is defined as:

$$\Pi(t) = \begin{cases} 1 & |t| \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

The NRZ format corresponding to the binary sequence 101101 is shown in Figure 1, along with other baseband digital formats.

The *return-to-zero* (RZ) format is characterized by an $a_n \in \{0, 1\}$ and $p(t) = A\Pi(\frac{t+D/4}{D/2})$ or any shifted version. Because the signal level returns to zero in the middle of the bit value, we should expect relatively higher frequency components in RZ compared with NRZ format, an undesirable effect. However, the RZ format aids the receiver in synchronization by guaranteeing a return to zero in the middle of the bit.

The polar NRZ format has the same pulse shape as the unipolar NRZ counterpart, but it uses an alphabet $a_n \in \{-1, +1\}$. Similarly, polar RZ is similar to unipolar RZ except that $a_n \in \{-1, +1\}$ for the polar case. These formats are shown in Figure 1 for the binary sequence 101101. Note that for the RZ and NRZ formats, for either polar or unipolar case, the signal has a nonzero DC value that is usually an undesirable characteristic. A popular digital baseband signal format is *Manchester encoding*, which guarantees zero DC value irrespective of the proportions of 1's and 0's in the digital signal. Manchester encoding format is also shown in Figure 1 along with other binary formats. All of these baseband digital signal formats are for the binary case where $D = T_b$—that is, the symbol period is the same as the bit period. In general, the symbol period will be greater than or equal to the bit period.

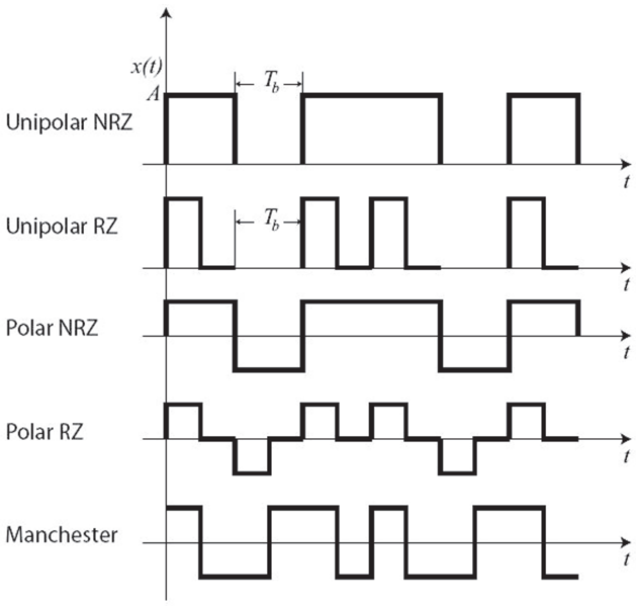

Figure 1: Digital PAM formats for unipolar NRZ, unipolar RZ, bipolar NRZ, bipolar RZ, and split-phase Manchester encoding for binary data 101101

Passband Signal Representation

The passband digital signal is obtained by modulating a carrier with a baseband digital signal. This process is referred to as *passband modulation*, and the carrier wave used is usually a sinusoidal wave. For communication over wireless channels and for longer distances, passband communication is used whereby passband signals are transmitted over the communication channel. Passband modulation is also employed whenever the communication medium is to be shared for multiple simultaneous communications over different frequency bands, as in standard commercial AM and FM radio. A large number of digital modulation schemes are used in practice (see section 1.5 of Xiong 2000). One of the most basic schemes for passband digital transmission is digital phase modulation in which the phase of the carrier is modified according to the digital baseband signal. In the rest of the chapter, we describe and analyze various DPM schemes with a particular focus on PSK. The reader is referred to Proakis (2000) and Anderson and Sundberg (1991) for details of more advanced DPM techniques. Phase shift keying is a special case of DPM in which the phase of the carrier is selected, in accordance with the digital symbol, at the beginning of the symbol interval. Thus, in PSK, phase discontinuities are observed at the start of *symbol intervals* (or *symbol boundaries*). In another type of DPM, continuous phase modulation, there are no phase discontinuities but the phase is allowed to vary throughout the symbol interval in a continuous manner.

Let us first introduce the mathematical representation of a passband digital signal. Any passband digital signal $x_c(t)$ is completely described by its in-phase and quadrature components and a carrier frequency. Mathematically, we write

$$x_c(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t$$

where $x_c(t)$ is the modulated signal or passband signal, $x_I(t)$ is the in-phase component of the passband signal, $x_Q(t)$ is the quadrature component of the passband signal, and f_c is the frequency of the sinusoidal carrier.

Using the famous Euler's equation $e^{j\theta} = \cos\theta + j\sin\theta$, we can write the above expression as

$$x_c(t) = \Re[x_B(t)e^{j2\pi f_c t}]$$

where $x_B(t)$ is called the baseband equivalent signal or the complex envelope of the passband signal and is given by

$$x_B(t) = x_I(t) + jx_Q(t)$$

The in-phase and quadrature components and the complex envelope are all low-pass signals. The passband signal can be completely described by knowing the carrier frequency and the complex envelope. We may further note that the power spectral density of the passband signal is completely obtained from the power spectral density of the complex envelope by knowing the carrier frequency (Carlson, Crilly, and Rutledge 2002).

Constellation Diagrams

In passband digital systems, such as those that use digital phase modulation or PSK, a symbol is sent as a digital signal from the transmitter to the receiver during every symbol interval. The signal sent in each interval is chosen from a set whose cardinality depends on the level of modulation. For binary modulation, for example, the signal sent in each interval is chosen from one of the two possible signals and represents a single bit of information. For quaternary modulation, the signal sent in each symbol interval represents two binary bits and is chosen from a set of four signals.

For passband digital modulation, the signal sent in each symbol is represented by the expression for $x_c(t)$ given in the preceding section on passband signal representation and can be considered as a weighted combination of two orthogonal basis functions $\phi_1(t)$ and $\phi_2(t)$. For phase shift keying, a possible set of such basis functions is:

$$\phi_1(t) = \sqrt{\frac{2}{\mathrm{T}}}\cos 2\pi f_c t$$
$$\phi_2(t) = -\sqrt{\frac{2}{T}}\sin 2\pi f_c t$$

where f_c is the frequency of each of the basis functions, and T is its reciprocal.

The digital passband signal transmitted in each interval is obtained by taking the weighted sum of $\phi_1(t)$ and $\phi_2(t)$, where the weights are obtained by selecting some appropriate values for $x_I(t)$and $x_Q(t)$. Note that the basis functions $\phi_1(t)$ and $\phi_2(t)$ chosen above are not only orthogonal to each other—that is, $\int_T \phi_1(t)\,\phi_2(t)\,\mathrm{dt} = 0$—but also have unit energy—that is, $\int_T \phi_1^2\,(t)\,\mathrm{dt} = \int_\mathrm{T} \phi_2^2\,(t)\,\mathrm{dt} = 1$. To find the projection of a signal point along the basis function $\phi_1(t)$, we may simply use

$$x_{c1,\phi_1} = \int_T x_{c1}(t)\phi_1(t)dt$$

where $x_{c_1}(t)$ is one of the two possible signals that can be transmitted within a symbol interval, and $x_{c_1,}\,\phi_1$ is the projection of $x_{c_1}(t)$ along the basis function $\phi_1(t)$.

We usually show the projections along the basis functions as a point in a plane where the axes represent the basis functions. Each point in such a two-dimensional plane refers to a signal in one symbol interval. The figure obtained by displaying all possible points is called a *constellation diagram*. Thus, a binary digital system has a constellation diagram with only two points, while the constellation diagram of a quaternary signal contains four points in the plane of basis functions (Wikipedia 2007, "Constellation"). Example constellation diagram for commonly used PSK schemes are given in subsequent sections.

GENERATION OF COHERENT PSK SIGNALS

In this section, we introduce the process of generating a coherent PSK signal. To this end, we first provide a mathematical representation of the PSK signals. To develop an understanding of PSK signals, we start with the simplest of the PSK signals: the *binary phase shift keying* (BPSK) signal, which is also known as a *binary antipodal signal*. Any reference to PSK would implicitly mean coherent PSK, unless stated otherwise.

Representation of BPSK

The BPSK signal is represented by two points on the constellation diagram. Corresponding to each constellation point, the two possible digital passband signals, one of which is selected for transmission in each symbol interval, are given by:

$$x_{c1}(t) = A\cos 2\pi f_c t$$
$$x_{c2}(t) = -A\cos 2\pi f_c t$$

In terms of the basis functions given in the preceding section on constellation diagrams, these two passband signals are

$$x_{c1}(t) = A\sqrt{\frac{T}{2}}\phi_1(t)$$
$$x_{c2}(t) = -A\sqrt{\frac{T}{2}}\phi_1(t)$$

The projection of $x_{c_1}(t)$ along the two axes in the constellation diagram is

$$x_{c1,\phi_1} = \int_\mathrm{T} x_{c1}(t)\phi_1(t)dt = A\sqrt{\frac{T}{2}}$$
$$x_{c1,\phi_2} = \int_\mathrm{T} x_{c1}(t)\phi_2(t)dt = 0$$

Similarly, the projection of $x_{c_2}(t)$ along the two axes in the constellation diagram is

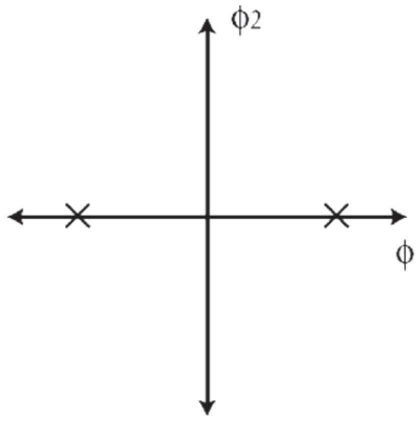

Figure 2: Constellation diagram for BPSK

$$x_{c2,\phi_1} = \int_T x_{c2}(t)\phi_1(t)dt = -A\sqrt{\frac{T}{2}}$$
$$x_{c2,\phi_2} = \int_T x_{c2}(t)\phi_2(t)dt = 0$$

The constellation diagram in Figure 2 shows $x_{c_1}(t)$ and $x_{c_2}(t)$ as points on the plane of basis functions. Both constellation points lie along the $\phi_1(t)$ axis with no component along $\phi_2(t)$. Furthermore, the two constellation points are 180 degrees from each other. The total number of constellation points (i.e., two in this case) indicate that the constellation diagram belongs to a binary digital system. For a quaternary system, the constellation diagram would contain four constellation points, as will be explained in later sections.

Every signal has an associated energy; therefore, each point in a signal constellation diagram has an associated transmitted energy. The average energy of all of the signal points in a constellation diagram is referred to as *transmitted energy per symbol*. For a binary constellation (i.e., with two constellation points), the transmitted energy per symbol is equal to the *transmitted energy per bit*, usually denoted by E_b. Thus, for BPSK signals, the energy of signal $x_{c_1}(t)$ within one symbol interval is

$$\begin{aligned} E_{x_{c1}} &= \left\langle x_{c1}(t)x_{c1}(t) \right\rangle \\ &= \int_T x_{c1}(t)c_{c1}(t)dt \\ &= \int_T \frac{A^2T}{2}\phi_1(t)\phi_1(t)dt \\ &= \frac{A^2T}{2} \end{aligned}$$

By symmetry, the energy of the signal $x_{c_2}(t)$ within one symbol interval is $E_{x_{c2}} = \frac{A^2T}{2}$. In general, the energy of a signal point is given by the sum of squares of projections along each basis function. Thus, for BPSK, the energy for the first constellation point is given by

$$\begin{aligned} E_{x_{c1}} &= \sqrt{x_{c1,\phi_1}^2 + x_{c1,\phi_2}^2} \\ &= \frac{A^2T}{2} \end{aligned}$$

Finally, the transmitted energy per bit for BPSK is given by averaging the energy per bit corresponding to each signal point in the constellation:

$$\begin{aligned} E_b &= \frac{\left\langle x_{c1}(t)x_{c2}(t) \right\rangle + \left\langle x_{c2}(t)x_{c2}(t) \right\rangle}{2} \\ &= \frac{A^2T}{2} \end{aligned}$$

Generation of BPSK

In practice, a binary digital signal consists of bits, a sequence of 1's and 0's, and one of the two signals in BPSK constellation is assigned to each bit. For example, for the BPSK signals given in the preceding section, a 1 can be represented by $x_{c_1}(t)$ while a 0 can be represented by $x_{c_2}(t)$. Thus, the transmitted signals that correspond to each bit value will be as shown in Figure 3. The assignment of bit values to one of the constellation points is completely arbitrary, and the transmitter and receiver must use the same assignment. For example, for BPSK representation in the previous section, one could assign $x_{c_1}(t)$ to bit value 0 and $x_{c_2}(t)$ to bit value.

A simple method to generate the BPSK signal is to take the polar NRZ baseband signal and multiply it with the sinusoidal carrier at frequency f_c, as shown in Figure 4. The resulting product will be a BPSK signal and is shown in Figure 5 for an example binary sequence 101101. The carrier frequency is deliberately chosen to be an integer multiple of the inverse of symbol period. It will be shown in the section on demodulation that this is not a strict requirement on the carrier frequency as long as it is much larger than the inverse of the symbol period.

Representation and Generation of QPSK

The *quadriphase shift keying* (QPSK) signal is represented by four points on the constellation diagram. As usual, for every symbol period, we select one from the four symbols for transmission during the same symbol period. However, in this case, each symbol represents two information bits and therefore the symbol period is twice the bit period. The four signals, each corresponding to one constellation point, are given by the generic expression for passband signals:

$$x_c(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t$$

To generate the QPSK signal, we need to find the in-phase and quadrature components, $x_I(t)$ and $x_Q(t)$, for each of the four signal points. These components are listed in Table 1.

Thus, for example, the expression for the signal $x_{c_1}(t)$ that corresponds to the first constellation point is

$$\begin{aligned} x_{c1}(t) &= \frac{A}{\sqrt{2}}\cos 2\pi f_c t - \frac{A}{\sqrt{2}}\sin 2\pi f_c t \\ &= \frac{A}{2}\sqrt{T}\phi_1(t) + \frac{A}{2}\sqrt{T}\phi_2(t) \end{aligned}$$

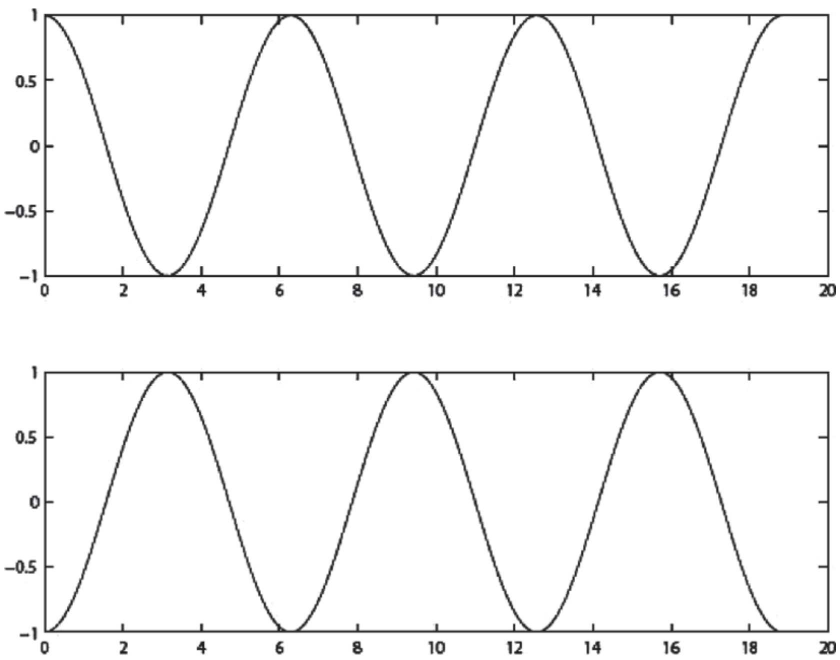

Figure 3: Signal set for BPSK with $A = 1$

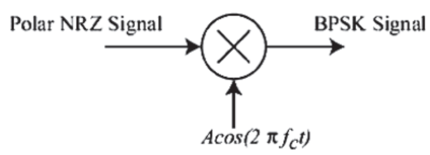

Figure 4: Generation of BPSK signal from polar NRZ signals

Then the projections of $x_{c_1}(t)$ on the axes representing the two basis functions are

$$x_{c1,\phi_1} = \int_T x_{c1}(t)\phi_1(t)dt = \frac{A}{2}\sqrt{T}$$
$$x_{c1,\phi_2} = \int_T x_{c1}(t)\phi_2(t)dt = \frac{A}{2}\sqrt{T}$$

The projections of other signal points on the basis functions can be similarly determined. The complete constellation diagram for the QPSK signal is shown in Figure 6.

For QPSK, one symbol represents two signal bits, which means that the energy per bit is one-half the energy per symbol. The energy per symbol can be determined by taking the average energy for each signal point in the constellation. For the first constellation point, the corresponding energy, if that symbol is transmitted in one symbol period, is given by

$$\begin{aligned} E_{x_{c1}} &= \langle x_{c1}(t)x_{c1}(t)\rangle \\ &= \int_T x_{c1}(t)x_{c1}(t)dt \\ &= \frac{A^2T}{2} \quad \text{(using orthonormality of basis functions)} \end{aligned}$$

This energy can also be determined by taking the sum of squares of projections along each basis function, as given below:

$$\begin{aligned} E_{x_{c1}} &= \sqrt{x^2_{c1,\phi_1} + x^2_{c1,\phi_2}} \\ &= \frac{A^2T}{2} \end{aligned}$$

By symmetry, the energy of other signal points within one symbol interval is $\frac{A^2T}{2}$ and thus the transmitted energy per symbol is $\frac{A^2T}{2}$. Finally, the energy per bit for QPSK is

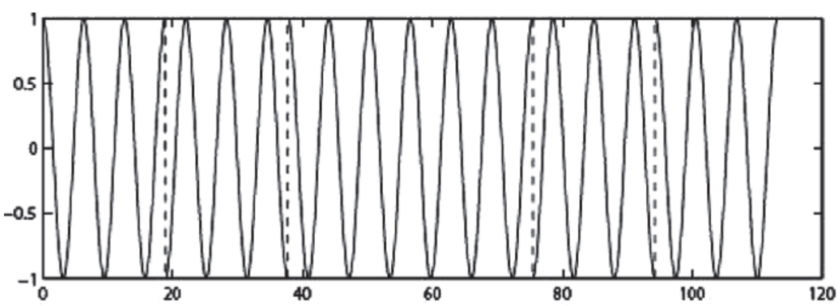

Figure 5: BPSK-transmitted signal for bit sequence 101101

Table 1: In-Phase and Quadrature Components of a QPSK Signal

Constellation point signal	In-phase component, $x_I(t)$	Quadrature component, $x_Q(t)$
$x_{c1}(t)$	$\frac{A}{\sqrt{2}}$	$\frac{A}{\sqrt{2}}$
$x_{c2}(t)$	$-\frac{A}{\sqrt{2}}$	$\frac{A}{\sqrt{2}}$
$x_{c3}(t)$	$-\frac{A}{\sqrt{2}}$	$-\frac{A}{\sqrt{2}}$
$x_{c4}(t)$	$\frac{A}{\sqrt{2}}$	$-\frac{A}{\sqrt{2}}$

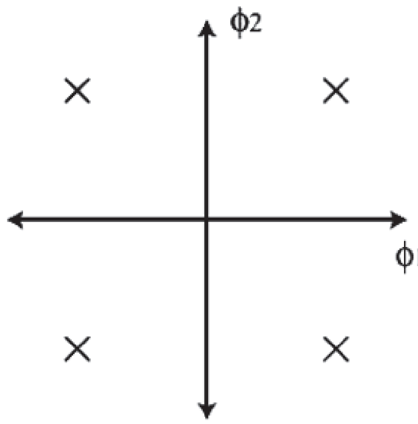

Figure 6: Constellation diagram for QPSK

$$E_b = \frac{1}{2}\frac{A^2T}{2} = \frac{A^2T}{4}$$

Now that we have expressions for each signal point in the QPSK constellation, the generation of the QPSK signal becomes straightforward. First, we emphasize that each constellation point represents two signal bits and, as in the case of BPSK, every possible pair of bits needs to be mapped onto one of the constellation points. Because there are four possible combinations a bit pair may take—00, 01, 10, and 11—we have just enough constellation points to map these bit pairs. Once again, the mapping is arbitrary as long as a single bit pair maps onto a single constellation point. Next, we note from Table 1 that in-phase and quadrature component pairs can be generated by mixing two polar NRZ BPSK signals. Thus, the QPSK signal can be generated by adding two BPSK signals that are generated using *quadrature carriers*. To generate the two BPSK signals, we split the polar NRZ encoded information bits into even and odd numbered bits using a demultiplexer and perform individual BPSK modulation as shown in Figure 7.

M-ary PSK

The constellation points in BPSK and QPSK signals may be considered as lying along a circle, where one constellation point can be obtained by adding some phase shift to another constellation point. The two constellation points in BPSK are 180 degrees apart, at 0 degrees and 180 degrees, whereas the constellation points in QPSK lie at 45, 135, 225, and 315 degrees along a circle whose radius can be computed from the projections of any point along the two basis functions. In both cases, the circle along which the constellation points lie has a radius equal to the square root of the transmitted energy per symbol.

The idea of placing the constellation points along a circle in the basis functions plane can be extended to higher modulation levels. For example, in 8-PSK (or 8-ary PSK) eight points are spaced at 45 degrees from adjacent points along a circle in the plane formed by the two basis functions, as shown in Figure 8. This also means that the signal energy of each constellation point is the same and is equal to the transmitted energy per symbol. For M-ary PSK, there are M constellation points along a circle in the basis function plane and each constellation point represents $\log_2 M$ bits transmitted in each symbol.

M-ary PSK signals can be generated by generalizing the idea of generating the QPSK signal from two BPSK signals. Theoretically, one signal point on the M-ary constellation can be obtained from another by adding appropriate phase shift to the modulated signal. Practically, every possible $\log_2 M$-bit sequence is mapped onto a constellation point. When a given bit sequence of length $\log_2 M$ bits occurs in the message stream, the signal corresponding to the mapped constellation point is transmitted

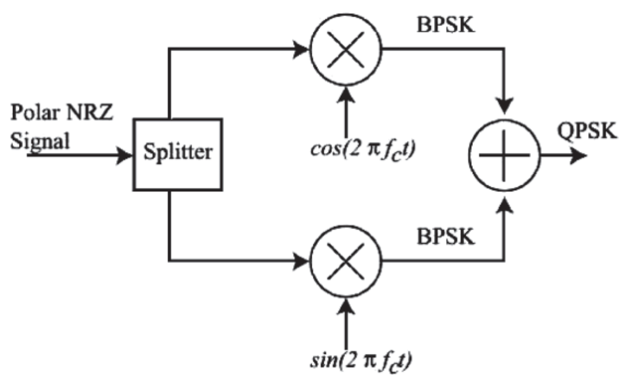

Figure 7: Generation of QPSK signal from polar NRZ signals

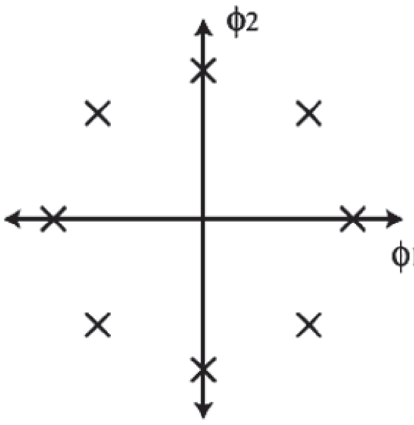

Figure 8: Constellation diagram for M-ary PSK

in the symbol interval. To generate the signal corresponding to a constellation point, the in-phase and quadrature components of the signal are summed after modulating with cos $2\pi f_c t$ and sin $2\pi f_c t$, respectively.

DEMODULATION OF COHERENT PSK SIGNALS

As mentioned in the early background section, coherent PSK is used when the receiver or the demodulator is able to locally generate the carrier with correct phase and frequency. To this end, the coherent receiver tracks the carrier by using a carrier recovery circuit. We will briefly describe the carrier recovery circuit in the later section on synchronization and carrier recovery while discussing synchronization and its effects on the performance of PSK signals. For this section, we assume that the demodulator has some mechanism to track the carrier and will somehow be able to generate the carrier with the correct frequency and phase. We will also assume that the transmitted signal does not encounter any imperfection or noise in the channel. In real communications, however, the received signal will not be an exact replica of the transmitted signal, only a noisy version of it. This will lead to occasional errors in making a decision at the demodulator about what was sent from the transmitter. For an understanding of the demodulation process, we may ignore the channel imperfections and later evaluate the effects of noise once the demodulation process is completely understood.

A simple construction of a coherent BPSK receiver is shown in Figure 9, where $r(t)$ is the received signal. Ignoring the noise, the received signal in each symbol interval

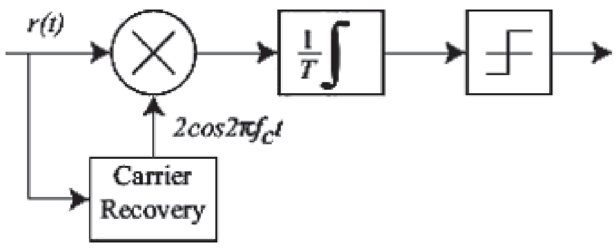

Figure 9: Demodulation of BPSK signals

is equal to the signal corresponding to one of the two constellation points in the BPSK constellation. Recall from the earlier section on BPSK that the two possible symbols in each symbol interval are:

$$x_{c1}(t) = A\cos 2\pi f_c t$$
$$x_{c2}(t) = -A\cos 2\pi f_c t$$

For a single symbol period, the received signal will thus be equal to either $xc_1(t)$ or $xc_2(t)$. In general, for $-\infty < t < \infty$, the received signal $r(t)$ can be written in terms of a polar NRZ signal using the previously discussed mechanism for generating a BPSK. In other words,

$$r(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT)\cos 2\pi f_c t$$

where $a_n \in \{-1, +1\}$ and $p(t) = A\Pi(\frac{t}{T})$. Thus, the signal $\hat{r}(t)$ at the output of the integrator is given by

$$\begin{aligned}
\hat{r}(t) &= \frac{1}{T}\int_T r(t)^2 \cos 2\pi f_c t dt \\
&= \frac{1}{T}\int_{(k-\frac{1}{2})T}^{(k+\frac{1}{2})T} \left(\sum_{n=-\infty}^{\infty} a_n A\Pi\left(\tfrac{t-nT}{T}\right)\cos 2\pi f_c t \right) 2\cos 2\pi f_c t dt \\
&= \frac{1}{T}\int_{(k-\frac{1}{2})T}^{(k+\frac{1}{2})T} Aa_k 2\cos^2 2\pi f_c t dt \\
&= \frac{Aa_k}{T}\int_{(k-\frac{1}{2})T}^{(k+\frac{1}{2})T} (1 + \cos 4\pi f_c t) dt \\
&= Aa_k + \frac{Aa_k}{4\pi T f_c}[\sin 4\pi f_c (k + \tfrac{1}{2})T - \sin 4\pi f_c (k - \tfrac{1}{2})T] \\
&= Aa_k + \frac{Aa_k}{4\pi T f_c} 2\cos 4\pi f_c kT \sin 2\pi f_c T
\end{aligned}$$

In the above expression, the second term is approximately equal to zero if $f_c T$ » 1, and is exactly zero if f_c is an integer multiple of 1/T. Thus, the input to the decision device, shown as the last box in Figure 9, is approximately (or exactly if f_c is an integer multiple of 1/T equal to Aa_k). The decision device infers that a $+1$ was sent from the transmitter if the input to the decision device is greater than 0 and decides that a -1 was sent otherwise. Thus, in the absence of noise, one can completely and perfectly recover the transmitted signal using the above demodulation scheme. The presence of noise in the signal would cause the input to the decision device to be different from Aa_k, depending on the magnitude of the noise, and may lead to occasional errors in inferring what was sent from the transmitter.

The demodulator for QPSK and M-ary PSK can be constructed on a similar principle, whereby the in-phase and quadrature components are demodulated separately and then combined to generate the transmitted signal (Xiong 2000).

SPECTRAL CHARACTERISTICS OF PSK SIGNALS

The *power spectral density* (PSD) of a modulated signal is a direct indication of the bandwidth efficiency of the modulation scheme. A modulated signal with *wider* PSD consumes more transmission bandwidth compared to a signal that has narrower PSD. We will provide expression for the power spectral density of M-ary PSK signals, which can be specialized for $M = 2$ (BPSK) and $M = 4$ (QPSK). To simplify things, we use the fact that for passband signals with certain properties, the PSD can be obtained by dividing by the symbol duration the power spectral density of the corresponding baseband signal. A BPSK signal generated from the polar NRZ signal in the previous section on BPSK possesses these properties and, therefore the PSD of the BPSK signal is simply 1/T times the PSD of the underlying polar NRZ signal shifted to baseband. Recall that the BPSK signal is given by

$$x(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT)\cos 2\pi f_c t$$

whose underlying baseband signal is the following polar NRZ signal:

$$\begin{aligned} x(t) &= \sum_{n=-\infty}^{\infty} a_n p(t - nT) \\ &= \sum_{n=-\infty}^{\infty} a_n A\Pi\left(\tfrac{t-nT}{T}\right) \end{aligned}$$

The energy spectral density of $x(t)$ in a single symbol period is given by the squared Fourier transform of $x(t)$ by considering just one symbol period. Thus, if $\tilde{x}(t)$ represents the single symbol of $x(t)$, then a possible value of $\tilde{x}(t)$ is $a_n\ A\Pi(\frac{t}{T})$. The Fourier transform $\tilde{X}(f)$ of $\tilde{x}(t)$ is, therefore, $a_n AT$ sinc fT. Thus, the *energy spectral density* (ESD) of the signal $x(t)$ is $a_n^2\ A^2T^2$ (sinc $fT)^2$; and because $a_n \in \{-1+1\}$, the ESD of $x(t)$ is A^2T^2 (sinc $fT)^2$. Then the PSD of the polar NRZ signal $x(t)$ is given by its ESD divided by the symbol period—that is,

$$\begin{aligned} \text{PSD of } x(t) &= S_x(f) = 1/T\, A^2T^2(\text{sinc}\, fT)^2 \\ &= A^2T(\text{sinc}\, fT)^2 \end{aligned}$$

Finally, the baseband-shifted PSD of BPSK signal is given by the PSD of the baseband polar NRZ signal divided by the symbol period T. Thus,

$$\begin{aligned} \text{Shifted PSD of BPSK} &= S_{x_c}(f) = 1/T\, A^2T(\text{sinc}\, fT)^2 \\ &= A^2(\text{sinc}\, fT)^2 \end{aligned}$$

It can be seen that the PSD of a polar NRZ signal falls off with squared frequency and crosses the first null at $fT = 1$ or $f = 1/T$. Thus, the PSD of a BPSK signal will also fall off with squared frequency, will be centered at f_c, and will hit the first null at $f_c \pm 1/T$. The PSD of a BPSK signal for the positive frequencies is shown in Figure 10.

The PSD of a QPSK signal can be easily determined by considering the structure that was used earlier (in the section on the representation and generation of QPSK) for the generation of BPSK by taking the sum of two independently generated BPSK signals. In other words, in one symbol period, we generate two independent BPSK signals, each of which has a corresponding baseband polar NRZ signal given by $a_n \frac{A}{\sqrt{2}} \Pi\left(\frac{t}{T}\right)$. Because this signal is $1/\sqrt{2}$ times the polar NRZ signal $\tilde{x}(t)$ considered above, its PSD is one-half of the PSD of the BPSK signal considered above. However, the QPSK is generated by summing up

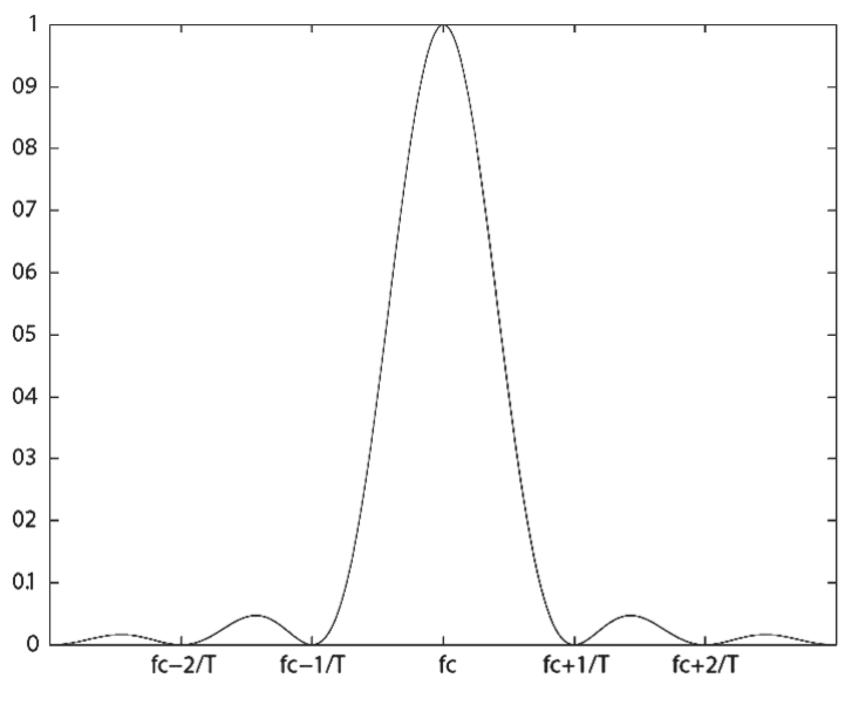

Figure 10: Power spectral density of BPSK signals

two such independent BPSK signals; thus, the PSDs are also added, and the PSD of the QPSK signal has the same expression as in the case of BPSK and is given below:

$$\begin{aligned}\text{Shifted PSD of QPSK} &= 2 \times (A/\sqrt{2})^2 (\text{sinc}\, fT)^2 \\ &= A^2 (\text{sinc}\, fT)^2\end{aligned}$$

This expression for power spectral density is also correct for an M-ary PSK modulation with parameter $A = \sqrt{2E/T}$, where E and T are energy per symbol and symbol period, respectively.

ERROR PERFORMANCE OF PSK SIGNALS

An important performance parameter of any digital modulation scheme is the average number of bits that are inferred incorrectly at the receiver. In an ideal communication channel, no errors are observed, but practical channels are noisy, which means that the signals transmitted through the channel are contaminated by noise. We usually model the noise as additive white Gaussian noise (Wikipedia 2007, "Additive") —that is, the probability density function of noise, which is assumed to be added to the transmitted signal, follows a Gaussian distribution, and individual noise samples are uncorrelated with each other, leading to flat power spectral density.

The effect of additive noise is to add an undesirable random parameter (voltage, phase, or frequency) to the actual signal. This results in a received signal that has moved away from the original constellation point used for that signal in that symbol period. For distinguishing the received signals corresponding to two neighboring constellation points, the receiver usually places the decision boundary midway between the two points. Thus, for the BPSK signal, where the two constellation points are at $\pm A\sqrt{T/2}$, the decision boundary is the vertical line midway between the two constellation points. Thus, received signals in the first and fourth quadrant are mapped to $\pm A\sqrt{T/2}$, while the signals in the second and third quadrant are mapped to $-A\sqrt{T/2}$. Similarly, for QPSK, the lines along the horizontal and vertical axes form the decision boundaries. An error results when the noise added to the signal is such that the contaminated signal crosses a decision boundary such that it is mistakenly regarded as belonging to a neighboring constellation point.

Although the effect of additive noise is simply to move the received signal away from its corresponding constellation point with the addition of an undesirable random parameter, the signals also undergo fading in wireless communication channels. Fading may increase the bit error rates by many times compared to those obtained in additive noise channels only (Rappaport 2001). In this chapter, we only include the bit error rates (or probability of bit errors) in *additive white Gaussian noise* while omitting the error rates for fading channels that can be found in more advanced texts (e.g., Proakis 2000; Barry, Lee, and Messerschmitt 2003).

To find the bit error rate (or error probability) of BPSK, we recall from the earlier section on the representation of BPSK that the BPSK transmitter sends one of the two possible symbols, $xc_1(t)$ or $xc_2(t)$, in each symbol interval. There are two possible ways a receiver can make an error: (1) $xc_1(t)$ was transmitted, but the additive noise pushed the received signal into the region that corresponds to $xc_2(t)$; or (2) $xc_2(t)$ was transmitted, but the additive noise pushed the received signal into the region that corresponds to $xc_1(t)$. For obtaining a quantitative expression, we consider additive noise with a constant two-sided power spectral density of $N_0/2$ and Gaussian distribution after demodulation, which is given by:

$$f_N(n) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{n^2}{N_0}}$$

The probability of error in a single bit interval is given by the sum of probabilities of two mutually exclusive events (Couch 1995):

1. The transmitter sent $xc_1(t)$ and the receiver detected $xc_2(t)$, which happens when additive noise after demodulation is less than $xc_{1,}\phi_1$.
2. xc_1The transmitter sent $xc_2(t)$ and the receiver detected $xc_1(t)$, which happens when additive noise after demodulation is greater than $-xc_{2,}\phi_1$.

If the probability of sending $xc_1(t)$ is the same as the probability of sending $xc_2(t)$, which is usually the case, then by symmetry the error probability is given by the probability that the demodulated noise is greater than $x_{c1,\phi 1} = A\sqrt{\frac{T}{2}} = \sqrt{E_b}$. Thus, the bit error probability $P_{b,\text{BPSK}}$ is given by

$$\begin{aligned}P_{b,\text{BPSK}} &= P\{\text{Additive Noise} > \sqrt{E_b}\} \\ &= \int_{\sqrt{E_b}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{n^2}{N_0}} dn \\ &= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\end{aligned}$$

where $Q(\cdot)$ is the tail of the standard Gaussian distribution (a Gaussian distribution with zero mean and unit variance).

A similar method of error probability analysis for QPSK can be carried out and is given in Haykin (2000). In summary, for QPSK, the symbol error probability is given by

$$P_{S,\text{QPSK}} = 2Q\left(\sqrt{\frac{E}{N_0}}\right)$$

where E is the symbol energy, and for QPSK the symbol energy is twice the bit energy as explained in the earlier section on BPSK representation—that is, $E = 2E_b$. With using a special encoding technique called *Gray encoding*, the probability of bit error in QPSK can be kept at one-half of the probability of symbol error. Thus, the bit error rate for QPSK is given by

$$\begin{aligned} P_{b,\text{QPSK}} &= \frac{1}{2} \times 2Q\left(\sqrt{\frac{E}{N_0}}\right) \\ &= Q\left(\sqrt{\frac{E}{N_0}}\right) \\ &= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \end{aligned}$$

which is the same as the bit error rate for BPSK. Thus, coherent QPSK delivers the same bit error rates as coherent BPSK but has the capability of transmitting data twice as fast, keeping other parameters constant. Thus, in practice, coherent QPSK is a preferred modulation scheme when compared with coherent BPSK (Simon, Hinedi, and Lindsey 1994).

SYNCHRONIZATION AND CARRIER RECOVERY

As we observed in the section "Demodulation of Coherent PSK Signals," coherent demodulation of coherent PSK requires the availability of a correct carrier frequency and phase at the receiver. This can be accomplished by using a carrier recovery circuit that usually employs a *phase-locked loop* (Xiong 2000). In fact, if the transmitter chooses to include an extra *pilot* signal at the carrier frequency (at which the PSK signals have no spectral components), the receiver may use this pilot signal to acquire the synchronization. If the pilot signal is not included in the transmitted signal, then carrier recovery is accomplished by using a nonlinear circuit. A newer approach to carrier synchronization is by setting up a DSP algorithm that maximizes the likelihood estimate of the carrier phase in an iterative manner (Haykin 2000).

Differential PSK does not require carrier synchronization because it can use noncoherent detector. However, both coherent and differential modulation require another type of synchronization called *symbol synchronization*. Note from the BPSK demodulation discussed earlier that the integral operation in the demodulator is carried out over precisely one symbol time. Therefore, the receiver must somehow determine the start and end of each symbol. Symbol synchronization, also referred to as *clock recovery*, is extremely important for correct detection of symbols and is usually implemented in DSP before performing carrier synchronization.

From the demodulation process, we note that the accuracy of clock recovery (or symbol synchronization) and phase synchronization determines the accuracy of the decisions taken at the receiver. It is shown in Xiong (2000) that a phase synchronization error of ϕ will result in a reduction in the amplitude of the signal that is fed to the decision device by a factor $\cos\phi$. Thus, the bit error probability of the BPSK signal in the presence of a phase error ϕ will be given by

$$P_b = Q\left(\sqrt{\frac{2E}{N_0}}\cos\phi\right)$$

From this equation, we note that the argument of the Q function is directly affected by the phase error, and thus even small local carrier phase errors may significantly affect the bit error rates of coherent BPSK systems.

SPECTRUM CONTROL IN DIGITAL PHASE MODULATION

Coherent PSK signals undergo abrupt phase transitions at the beginning of symbol intervals, causing high-powered spectral side lobes that may interfere with adjacent channels. To illustrate the abrupt phase transitions, let us consider the example of QPSK signal described earlier in "Representation and Generation of QPSK." The QPSK constellation shown in Figure 6 consists of four points, exactly one of which is transmitted in one symbol period, corresponding to a two-bit sequence that appears in the bit stream of message signal during that symbol period. The signal transmitted in one symbol interval does not depend on the signal transmitted in a previous or future symbol interval. Thus, the constellation point in each symbol interval is chosen independently. Because the constellation points represent signals with differing phases (at 45, 135, 225, and 315 degrees), phase change at each symbol boundary could be 0 degrees, ±90 degrees. and ±180 degrees.

Another issue related to abrupt phase transitions turns up when we consider the demodulator that invariably uses a bandpass filter before detection in order to limit the amount of additive white Gaussian noise. Some PSK transmitters will also include the bandpass filter to limit the side lobe power and consequent interference with adjacent frequency bands. Passing a PSK signal through a bandpass filter causes amplitude variation through a phenomena called FM-to-AM conversion (Carlson, Crilly, and Rutledge 2002). This amplitude variation can deteriorate the error rate performance of the overall system.

To combat abrupt phase transitions at the beginning of symbol intervals, variations of PSK have been devised. One such variation for QPSK is called *staggered QPSK* or *offset-keyed* (or *offset*) *QPSK*, abbreviated OQPSK. Using OQPSK, constellation points chosen in any two consecutive symbol periods are not allowed to be diagonally opposite to each other, thus avoiding the ±180-degree phase transition at the beginning of the symbol period. Thus, the allowable phase transitions at the beginning of a symbol period are limited to 0 degrees and ±90 degrees. OQPSK is generated by delaying the bit stream responsible for generating the quadrature component by half a symbol period. Although OQPSK exhibits the same bit error performance as exhibited by QPSK, it results in reduced amplitude fluctuations after bandpass filtering when compared with QPSK.

The use of OQPSK limits the phase transitions to at most ±90 degrees at the beginning of a symbol interval. These phase transitions can be eliminated altogether by using CPM. In CPM, the phase transitions, which happen at the beginning of a symbol interval in case of QPSK and OQPSK, are spread over the whole symbol interval in such a way that a phase continuity is maintained throughout the symbol period, including the symbol boundaries. A generic continuous phase modulated signal is given by

$$x_c(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \phi_0 + \phi(t))$$

where the variable portion of phase $\phi(t)$ is given by

$$\phi(t) = 2\pi h \sum_{j=0}^{n} a_j q(t - jT)$$

and where $q(t)$ is called the phase-shaping pulse and h is the modulation index. The phase-shaping pulse $q(t)$ is related to the frequency-shaping pulse $g(t)$as:

$$q(t) = \int_{-\infty}^{t} g(y)dy$$

Note that the expression of CPM signal $x_c(t)$ indicates that the CPM is a variation of frequency modulation; however, because of a direct relationship between frequency and phase (instantaneous frequency is the derivation of instantaneous phase), a CPM signal can either be viewed as a frequency-modulated or a phase-modulated signal. When $g(t)$ is a rectangular pulse of duration LT, we get a special family of CPM called L-REC CPM. For $L = 1$,we get 1-REC CPM, which is also known as *continuous phase frequency shift keying*. A further specialization of CPFSK with $h = 21$ yields the well-known *minimum shift keying* (MSK). For demodulation purposes, a usual way to interpret a CPM signal is to consider it as a concatenation of a trellis code followed by memoryless modulation (Wilson 1995). Thus, the demodulation of CPM signals may be carried out by a linear demodulator followed by a sequence detector.

CONCLUSIONS

Digital phase modulation is a widely used modulation technique in computer and communication networks. This type of modulation has an attractive property—it maintains a constant envelope—and is therefore suitable for communication in nonlinear channels. Many existing communication networks use phase shift keying or other digital phase modulation schemes for communication at the physical layer. Specifically, current wireless LAN standards such as IEEE 802.11b, IEEE 802.11g and others such as Bluetooth and GSM make use of digital phase modulation in various transmission modes.

We studied the methods for generating and demodulating the PSK signals and also learned PSK signal representation in the signal space. We evaluated the PSK schemes with respect to the error probabilities they offer and briefly described the two types of synchronization required in coherent systems. Expressions for bit error rates for BPSK and QPSK are provided in this chapter, and we learned that, keeping other parameters same, QPSK delivers data at twice the rate of BPSK and therefore is a preferred modulation technique.

Phase shift keying is the simplest form of digital phase modulation. Constant envelope is the most notable and useful property of PSK signals. In general, all DPM schemes maintain a constant envelope. Furthermore, PSK undergoes abrupt phase changes at symbol boundaries, resulting in relatively high energy at higher frequency side lobes. This abrupt phase change can be subsided by using techniques such as offset QPSK, which limits the phase jump at symbol boundaries. Furthermore, CPM allows a complete elimination of abrupt phase changes at symbol boundaries by allowing a continuous change of phase throughout the symbol. CPM signals may be viewed as coding followed by modulation and therefore may be demodulated by using a coherent demodulator followed by a sequence detector. By avoiding abrupt phase changes, CPM avoids high power in side lobes, resulting in spectrally efficient modulation.

GLOSSARY

Bit error rate: The average number of bits received in error when a digital signal is sent from one point to another.

Message signal: An electrical signal, typically a representative of a physical quantity, that is usually transmitted from the source to the destination in a communication system.

Carrier signal: A signal or wave that is a relatively high-frequency signal, usually a sinusoidal. One property of a carrier signal—phase, frequency, or amplitude—is varied according to a message signal that needs to be transmitted from one point to another.

Constellation: The representation of signal points of a digital modulation scheme on a plane whose axes are represented by orthonormal basis functions.

Continuous phase frequency shift keying (CPFSK): A special case of CPM in which the frequency-shaping pulse is rectangular with a duration equal to the symbol interval.

Continuous phase modulation (CPM): A digital modulation in which the modulated signal is constrained to maintain a constant envelope and continuous phase, avoiding abrupt changes in phase.

Demodulation: The process of extracting the message signal from the modulated signal received at the receiving end of a communication system.

Minimum Shift Keying (MSK): A special case of CPFSK. In particular, MSK is CPFSK with a minimum modulation index that results in orthogonal signaling.

Modulation: The process of varying some property (amplitude, phase, or frequency) of a carrier signal according to a message signal that needs to conveyed to the receiver.

Phase shift keying (PSK): A modulation method in which the phase of a carrier signal is varied directly in accordance with a digital message signal.

Synchronization: The process in which two quantities are made to vary in unison. In PSK, two types of synchronization are needed: phase synchronization and symbol synchronization.

CROSS-REFERENCES

See *Minimum Shift Keying (MSK)*; *Optical Fiber Communications*.

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