1. Prove the following relationship by induction:

2. An application requires creating medical images from m data points collected by the imaging system. The running times of two algorithms A and B as a function of the data points are:

   \[ T(m) = 2m \]

   \[ T(m) = 28800m \]

   For what values of m is algorithm A faster than algorithm B?
3. The following program fragment is intended to calculate the sum of odd and even integers up to n:

```c
scanf("%d", &n);
oddsum = 0;
evensum = 0;
for(j=0; j<=n; j++){
    if(j % 2 != 0)
        oddsum = oddsum + j;
    else
        evensum = evensum + j;
}
printf("%d \n", oddsum);
```

(a) Calculate the average running time T(n) of the above fragment

Assume that it takes one time unit each to do a scan, print, assignment, and testing a condition. Clearly show your work. You may mark the code above to show your work.

(20)
4. The running time of a program is found to be
   \[ T(n) = n + 6n + 2 \]

   (a) To prove that \( T(n) \) is \( O(n) \), is it possible to chose a pair of witnesses \( (n, c) \) such that \( n = 0 \)? Justify your answer.
5. The following program fragment computes the sum of integers in array X[0...p-1]:

\[
\begin{align*}
\text{sumX} &= 0; \\
\text{for} (i=0; i<p; i++) & \quad \text{sumX} = \text{sumX} + X[i];
\end{align*}
\]

Find an appropriate loop invariant and use it to prove that the above fragment works as intended.

(25)
6. (a) Write a recursive function findseq that returns TRUE if two integers x and y occur in sequence in a list L and FALSE otherwise. For example if L = (2, 10, 14, 5, 1, 3) then the function call findseq(1,2,L) should return FALSE whereas findseq(5,1,L) should return TRUE. Assume that the list L is implemented as a linked list.

(b) Write a function multiseq that returns an integer value indicating the number of times two integers x and y occur in sequence in a list L, assuming that L is implemented as an integer array. For example if L = (1,2,4,6,3,2,4) then the function call multiseq(2,4, ...) will return 2. Assume that a list can have a maximum of 1000 elements.