13.4. Bode Plots

A Bode plot is a plot of log-gain vs frequency and phase angle vs frequency on a log-frequency scale.

For a network function $H(w) = |H|/\Phi$

Gain in decibels (dB) = $20 \log_{10} |H|$

Example:

$$H(w) = \frac{1}{(j\omega/\omega_0) + 1} = |H|/\Phi$$

where, $Gain = |H| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$ and

phase angle = $\Phi = -\tan^{-1}(\omega/\omega_0)$

To find the gain in dB,

$$Gain\ in\ dB = 20 \log_{10} |H| = 20 \log_{10} \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

$$= 20 \log_{10}(1) - 20 \log_{10} \sqrt{1 + (\omega/\omega_0)^2}$$

$$= 0 - 20 \log_{10} \sqrt{1 + (\omega/\omega_0)^2}$$

For small frequencies, $\omega \ll \omega_0$, $\frac{\omega}{\omega_0} \ll 1$

so $Gain\ in\ dB = -20 \log_{10} 1 = 0$
For large frequencies, \( \omega \gg \omega_0 \), \( \left( \frac{\omega}{\omega_0} \right)^2 \gg 1 \)

\[
\text{Gain (dB)} = -20 \log \left( \frac{\omega}{\omega_0} \right)^2 = 20 \log \omega_0 - 20 \log \omega
\]

Asymptotic Curve:

\[
|H| = \begin{cases} 
0, & \omega < \omega_0 \\
20 \log \omega_0 - 20 \log \omega, & \omega > \omega_0 
\end{cases}
\]

\[\begin{array}{c|c|c|c}
\text{Gain (dB)} & 0 & -10 & 10 \\
\hline
0.1 \omega_0 & 0 & -10 & 0 \\
\omega_0 & 20 & -40 & -40 \\
10 \omega_0 & 40 & -60 & -60 \\
\end{array}\]

At \(\omega = \omega_0\), gain = -3 dB

\[\begin{array}{c|c|c|c}
\phi \text{ (degrees)} & 0 & -45 & -90 \\
\hline
0.1 \omega_0 & 0 & -45 & -90 \\
\omega_0 & -20 & -60 & -100 \\
10 \omega_0 & -40 & -80 & -120 \\
\end{array}\]

Note:

\[
\text{gain (dB)} = 20 \log \omega_0 - 20 \log \omega
\]

represents a straight line on a gain (dB) vs \(\omega\) (log-scale) with a slope of -20 dB for every ten times increase in frequency.

At \(\omega_1\), gain (dB) = \(20 \log \omega_0 - 20 \log \omega_1 = X\) dB

At \(10 \omega_1\), gain (dB) = \(20 \log \omega_0 - 20 \log (10 \omega) = (X + 20) \log 10 = (X - 20)\) dB
\( \omega_0 \) is also called the Break Frequency or Corner Frequency.

Example:

\[
H(\omega) = \frac{V_o}{V_s} = \frac{R + j \omega L}{R_s + R + j \omega L}
\]

To get Bode plot, write \( H(\omega) \) in the following form:

\[
H(\omega) = k \frac{1 + j (\omega/\omega_1)}{1 + j (\omega/\omega_2)}
\]

\( \omega_1, \omega_2 \) are corner frequencies.

Corner frequencies in the numerator are called zeros, and corner frequencies in the denominator are called poles.

So, \( \omega_1 \) is a zero, and \( \omega_2 \) is a pole.

\( k = \text{dc gain}, \) since \( k = \lim_{\omega \to 0} H(\omega). \)

Equation (1) can be written in the form of (2) as follows:

\[
H(\omega) = \left( \frac{R}{R + R_s} \right) \frac{1 + j \frac{\omega L}{R}}{1 + j \frac{\omega L}{R + R_s}}
\]

So, dc gain = \( \frac{R}{R + R_s} \); \( \omega_1 = \frac{R}{L} \); \( \omega_2 = \frac{R + R_s}{L} \)

Also \( \omega_2 > \omega_1 \), as it should be.
From (3) we can write
\[ |H(\omega)| = k \frac{\sqrt{1 + (\omega/\omega_i)^2}}{\sqrt{1 + (\omega/\omega_2)^2}} \]
where \( k, \omega_i \), and \( \omega_2 \) are given in (4), (5), and (6).

For asymptotic plot,
\[ |H(\omega)| = \begin{cases} 
  k \omega < \omega_i \\
  k(\omega/\omega_i); \quad \omega_i < \omega < \omega_2 \\
  k(\omega/\omega_2); \quad \omega_2 < \omega
\end{cases} \]

In dB
\[ 20 \log |H(\omega)| = \text{Gain (dB)} = \begin{cases} 
  20 \log k \quad \omega < \omega_i \\
  (20 \log k - 20 \log \omega_i) + 20 \log \omega; \quad \omega_i < \omega < \omega_2 \\
  (20 \log k - 20 \log \omega_2) + 20 \log \omega_2; \quad \omega_2 < \omega
\end{cases} \]

The plot is shown here:

The graph shows a linear segment with a slope of 20 dB/decade.

\[ 20 \log (H/k) = \begin{cases} 
  0 \quad \omega < \omega_i \\
  20 \log \omega - 20 \log \omega_i; \quad \omega_i < \omega < \omega_2 \\
  20 \log \omega_2 - 20 \log \omega_2; \quad \omega_2 < \omega
\end{cases} \]
\[ \phi = \mathcal{L}_k + \mathcal{L}_{1+j\left(\frac{\omega}{\omega_1}\right)} - \mathcal{L}_{1+j\frac{\omega}{\omega_2}} \]

\[ = 0 + \tan^{-1}\frac{\omega}{\omega_1} - \tan^{-1}\frac{\omega}{\omega_2} \]

READ EXAMPLES 13.4-1
AND 13.4-2 / PAGES 594-595.

Please see Figure 13.4-4 / P593
for the phase
Bode plot.