CS 431 Quiz 3 Solution

1. (6 points) Construct a simple belief network for troubleshooting computer bootup problems.
   a. (2 points) List the variables and their domains (list at least 5 variables including \textit{OSLoads} with a value of \textit{true} or \textit{false}).

   All are Boolean random variables with possible values of true or false.
   Keyb? = is the keyboard plugged in and/or working?
   Mouse? = is the mouse plugged in and/or working?
   Monitor? = is the monitor plugged in and/or working?
   Power? = is the power cord plugged properly and/or electricity is on
   CStarts? = is the computer on?
   HwProb? = is the hardware malfunctioning?
   MemProb? = is the memory bad?
   VideoProb? = is the graphics card not booting?
   DiskProb? = is the hard disk working?
   BIOSRuns? = is the BIOS running properly?
   Corruption? = is OS file(s) corrupted?
   OSHfNAR? = OS hangs for no apparent reason? - is OS Windows95/98/Me (a little MS bashing doesn’t hurt!)
   OSLoads? = is OS loaded and running properly?

   b. (4 points) Draw the belief network that \textit{compactly} represents the causal relationships among the variables.
2. (4 points) Consider the following belief network:

\[
\begin{array}{c}
A \\
\downarrow \\
C \\
\downarrow \\
D \\
\end{array}
\]

where \(A, B, C\) and \(D\) are Boolean random variables. Given \(P(A) = 0.2, P(B) = 0.5\) and the CPTs (\(T = \text{true}, F = \text{false}\)):

<table>
<thead>
<tr>
<th>(A)</th>
<th>(B)</th>
<th>(P(C \mid A \text{ and } B))</th>
<th>(C)</th>
<th>(P(D \mid C))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T)</td>
<td>(T)</td>
<td>0.8</td>
<td>(T)</td>
<td>0.4</td>
</tr>
<tr>
<td>(T)</td>
<td>(F)</td>
<td>0.5</td>
<td>(F)</td>
<td>0.8</td>
</tr>
<tr>
<td>(F)</td>
<td>(T)</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>(F)</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Compute the conditional probabilities \(P(D \mid B = \text{true})\).

By the product rule, normalization, conditional independences, and marginalization:

\[
P(D = \text{true} \mid B = \text{true}) = \alpha P(D \text{ and } B) = \alpha P(B) \sum_a \sum_c P(a) P(c \mid a, B) P(D \mid c)
\]

\[
= \alpha(0.5) \left\{ 0.2 \left[ (0.8)(0.4) + (0.2)(0.8) \right] + 0.8 \left[ (0.4)(0.4) + (0.6)(0.8) \right] \right\}
\]

\[
= \alpha(0.5) \left\{ (0.2)(0.48) + (0.8)(0.64) \right\} = \alpha(0.304)
\]

Similarly,

\[
P(D = \text{false} \mid B = \text{true}) = \alpha P(D = \text{false} \text{ and } B) = \alpha P(B) \sum_a \sum_c P(a) P(c \mid a, B) P(D = \text{false} \mid c)
\]

\[
= \alpha(0.5) \left\{ 0.2 \left[ (0.8)(0.6) + (0.2)(0.2) \right] + 0.8 \left[ (0.4)(0.6) + (0.6)(0.2) \right] \right\}
\]

\[
= \alpha(0.5) \left\{ (0.2)(0.52) + (0.8)(0.36) \right\} = \alpha(0.196)
\]

\(P(D \mid B = \text{true}) = \{0.608, 0.392\}\)