Computer Vision Fundamentals:
Homework Module 2
Due Tuesday, 29th October, (at the beginning of class)

1. Exercise 3.6 (Morphology)
2. Exercise 3.8 (Morphology)
3. Exercise 3.10 (Region Properties)
4. Exercise 3.11 (Region Properties)
5. Exercise 3.12 (Morphology / Region Properties)
6. Circularity Measures:

All four shapes have approximately the same area (verify). Using \( P_8 \) definition of perimeter, find \( C_1 \) circularity measure for each of the four shapes. Compare \( C_1 \) for circle with its ideal value. Order the shapes in descending order of \( C_2 \) by either roughly estimating \( C_2 \) by argument, or by computing it numerically using a statistical routine.

7. Connected Component Labeling: Consider the following binary image:
In each white square write a unique number from 1 to 28, based on the order in which it will be labeled during recursive connected components algorithm. Assume that the algorithm scans the image from left to right, top to bottom, and that the eight neighbors are checked in clockwise direction from North; i.e. in the order N, NE, E, SE, S, SW, W, NW.

8. Connected Component Labeling: Consider the following binary image:

![Image](image.png)

How many times will the recursive connected component labeling algorithm look at (read/write) each of the pixels labeled as 1, 2 and 3? Based on this, can you derive a general formula for total pixel read/write operations for a square object of $n$-by-$n$ pixels?

9. Derive the expression for a 2D Laplacian of Gaussian (LoG) filter (using definition of Laplacian operator) and show that the following 3x3 mask approximates it for small $\sigma$ (Hint: try $\sigma$ close to 0.5):

<table>
<thead>
<tr>
<th>0</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

10. Exercise 5.10 (LoG Edge Detector)

11. Exercise 10.7 (Canny’s Edge Detector)

12. Rotation Invariance of Gradient Magnitude: Show that the gradient magnitude is rotation invariant. Let the gradient at point $(x, y)$ be given by $(f_x, f_y)$. Assume that the object is rotated around Z-axis, and the point $(x, y)$ moves to $(x', y')$. Show that $\sqrt{f_x'^2 + f_y'^2} = \sqrt{f_x^2 + f_y^2}$

13. (Generalized Hough Transform). Discuss how the R-Table in GHT changes as a shape is (i) rotated, (ii) scaled. Based on this discussion, argue why rotation & scaling invariant GHT algorithm (discussed in class) makes sense.