Solution to Quiz 1 (12 points)

1(a) (2 points) **Pixel Coordinate Frame I** is the coordinate frame used to represent images in **row, column** format. The origin is taken to be at the top left corner of the image. This is a **discrete** coordinate system, where coordinates can only have **integer** values.

1(b) (2 points) **Real Image Coordinate Frame F** is the coordinate system of the image relative to some world coordinate system. It is in the same units as the world coordinate frame (e.g. **inches, mm**). Real world objects are mapped to this frame using the **camera model** (e.g. perspective, orthographic). This is a **real-valued** coordinate system. Typically the origin is where the optical axis intersects the image plane.

1(c) (2 points) Differences between **I** and **F** are:
- **I** is discrete and **F** is continuous.
- **I** represents the image in row/column format, whereas **F** represents the image in real-world coordinates, relating the image to the world through the camera matrix.

**I** and **F** are related to each other through scaling and translation. Coordinates in **F** need to be scaled by the ratio of the total number of pixels to the size of the CCD array, and then translated by half the number of pixels (to move the origin to the top left corner. Suppose the size of the CCD array is 2 x 2 cm and the image formed is 200 x 200 pixels, then

\[
I = \text{round} \left( \begin{bmatrix} 200 \ 0 \\ 200 \ 2 \end{bmatrix} F + \begin{bmatrix} 100 \\ 100 \end{bmatrix} \right)
\]

2(a) (3 points)
The given transformation is a shear along x-axis.

2(b) (3 points)
Shear along x-axis has transformation matrix of the form
\[
\begin{bmatrix}
1 & k \\
0 & 1
\end{bmatrix}
\]
with zero translation.

For the given transformation, the point (0,b) has moved to (a/5, b). This means that
\[
\begin{bmatrix}
1 & k \\
0 & 1
\end{bmatrix} \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} a/5 \\ b \end{bmatrix}
\]
This shows that \( kb = a/5 \), or \( k = a/5b \). Thus, the homogeneous affine transformation matrix for the transformation given in the figure is

\[
T = \begin{bmatrix}
1 & a/5b & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

It can be easily verified that the above transformation when applied to point \([a, b]^T\) (i.e. bottom right corner) yields \([a + a/5, b]^T\), which is correct with respect to the figure shown.