Quiz Number 2 – Solutions

Closed Book; Closed Notes; Time Given=10 minutes (Sep. 20, 2005)

“I certify that I have neither received nor given unpermitted aid on this examination and that I have reported all such incidents observed by me in which unpermitted aid is given.”

Name ______________________  Student ID __________________  Signature ______________________

Question 1: [5 points] Find the Fourier Transform of the function shown below. Try to express your answer in terms of sinc functions:

Solution:

First note that the independent variable of the given function is not significant when finding the Fourier transforms. Similarly, the answer can be specified in terms of any independent variable. Next, note that the given function is 2rect(\( f \)) \( \ast \) rect(\( f \)). Since rect(\( f \)) \( \leftrightarrow \) sinc(\( z \)), the required transform is 2sinc\(^2\)(\( z \)), by using the convolution property of Fourier transforms.

\[
X(f) = 2
\]

\[
-1 \quad \quad 1 
\]

\[
f
\]

\[\text{Solution:}
\]

Question 2: [5 points] Consider a linear time-invariant (LTI) system with impulse response \( h(t) \) whose Fourier transform is \( H(f) \). If the input \( x(t) \) to the LTI system is:

\[
x(t) = \sin 2\pi f_1 t + \cos 4\pi f_2 t
\]

Find the output \( y(t) \) of the LTI system.

Solution: First note that the input signal can be expressed in terms of exponentials, which are the eigenfunctions of LTI systems. Thus,

\[
x(t) = \sin 2\pi f_1 t + \cos 4\pi f_2 t
\]

\[
= \frac{1}{2j}(\exp^{j2\pi f_1 t} - \exp^{-j2\pi f_1 t}) + \frac{1}{2}(\exp^{j2\pi f_2 t} + \exp^{-j2\pi f_2 t})
\]

\[
\Rightarrow y(t) = \frac{1}{2j}(H(f_1) \exp^{j2\pi f_1 t} - H(-f_1) \exp^{-j2\pi f_1 t})
\]

\[
+ \frac{1}{2}(H(2f_2) \exp^{j2\pi f_2 t} + H(-2f_2) \exp^{-j2\pi f_2 t})
\]

For real \( h(t) \), \( H(f) \) is symmetric, and hence,

\[
y(t) = H(f_1) \sin 2\pi f_1 t + H(2f_2) \cos 4\pi f_2 t
\]