Midterm Examination-I

Open Book, Open Notes;
Calculators Allowed
Time Allowed = 2 hours (Oct. 01, 2005)

“I certify that I have neither received nor given unpermitted aid on this examination and that I have reported all such incidents observed by me in which unpermitted aid is given.”

Signature ________________________________

Name ________________________________ Student ID ________________________________

Problem 1 ___________ [18]
Problem 2 ___________ [10]
Problem 3 ___________ [20]
Problem 4 ___________ [12]
Problem 5 ___________ [15]
Problem 6 ___________ [12]
Problem 7 ___________ [28]
Problem 8 ___________ [25]
Problem 9 ___________ [20]

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TOTAL ___________ [160]
Problem 1: [18 points] A signal $x(t) = \text{rect}\left(\frac{t-1}{3}\right)$ is input to a filter whose impulse response is $h(t) = \text{rect}\left(\frac{t-3}{2}\right)$. What is the output $y(t)$ of the filter? Make sure to sketch and label the output.
Problem 2: [10 points] Evaluate $\int_{-\infty}^{\infty} [\text{sinc}(t) \star \text{sinc}(2t)] dt$, where $\star$ denotes the convolution operation.
Problem 3: [20 points] Consider two time domain functions, \( f(t) \) and \( g(t) \). We are interested in finding if the composite function \( f(g(t)) \) is even, odd, or neither. Determine this for the following cases (for each case, your answer should be even, odd, or neither. justify your answers; almost no credit will be given without justification. **Hint:** A signal \( h(x) \) is even if \( h(-x) = h(x) \) and is odd if \( h(-x) = -h(x) \):

(a) [5 points] \( f(t) \) is odd and \( g(t) \) is even.

(b) [5 points] \( f(t) \) is odd and \( g(t) \) is odd.

(c) [5 points] \( f(t) \) is even and \( g(t) \) is even.

(d) [5 points] \( f(t) \) is even and \( g(t) \) is odd.
Problem 4: [12 points] A raised cosine pulse is frequently used in digital communications, and is given by:

\[ r(t) = \text{rect}\left(\frac{t}{2}\right)(1 + \cos \pi t) \]

Find the Fourier transform of raised cosine pulse by direct integration. Simplify your answers expressing them in terms of sinc functions.
Problem 5: [15 points] A radar can measure the distance to a target by transmitting a waveform, receiving the reflected waveform back from the target and cross-correlating the transmitted and received waveforms. A maximum value of cross-correlation indicates the amount of delay between the transmitted and received waveforms. Let \( x(t) \) is the transmitted signal shown below, and \( y(t) \) is the received signal also shown below.

\[ x(t) \quad \text{1 volt} \quad 1 \quad 2 \quad 6 \quad 9 \quad t, \, \mu s \]

\[ y(t) \quad 0.5 \text{ volt} \quad 11 \quad 12 \quad 16 \quad 19 \quad t, \, \mu s \]

![Figure 1: Transmitted and Received Signal from a Radar](image)

(a) [10 points] For what value of \( \tau \) is \( R_{xy}(\tau) \) a maximum? (Use the rest of the page and next page for working; write your final answer here).

(b) [5 points] If the time units used in the figure are microseconds, how far is the target from the radar? You may assume that the electromagnetic signal propagates with the speed of light.
Problem 6: [12 points] Consider two baseband signals $x_1(t)$ and $x_2(t)$ having bandwidths $B_1$ and $B_2$ Hertz, respectively. We need to transmit $y(t) = x_1(t) \cdot x_2(t)$ through a digital communication system. The transmitter must sample $y(t)$ before further processing. The goal is to correctly recover $y(t)$ at the receiver, at least when the noise can be neglected.

(a) [6 points] Is there a minimum limit on the separation between the samples taken from $y(t)$? If yes, what is it? If not, why?

(b) [6 points] Is there a maximum limit on the separation between the samples taken from $y(t)$? If yes, what is it? If not, why?
Problem 7: [28 points] For each of the following systems, output \( y(t) \) is given in terms of input \( x(t) \). State whether these systems are linear or non-linear. Also state whether these systems are time invariant or time variant?

(a) [4 points] \( y(t) = x(t) \cos 2\pi ft \)

(b) [4 points] \( y(t) = \sin[x(t)] \)

(c) [4 points] \( y(t) = \int_{-\infty}^{\infty} x(\tau)e^{-j2\pi ft}\,d\tau \)
(d) [4 points] $y(t) = \frac{d}{dt} x(t)$

(e) [4 points] $y(t) = \cos[2\pi f t + x(t)]$

(f) [4 points] $y(t) = x(-t)$

(g) [4 points] $y(t) = \text{sgn}(x(t))$
Problem 8: [25 points] Consider a signal $x(t)$, with average value $\langle x(t) \rangle = 0$, and $S_x = \langle x^2(t) \rangle = \frac{1}{3}$. The signal $x(t)$ has a maximum value 0.8 and a minimum value -0.8. Suppose we need generate an AM signal $x_c(t) = A_c(1 + \mu x(t)) \cos 2\pi f_c t$ for transmission from an AM radio station, whose listeners would use standard envelop detectors.

(a) [5 points] What is the maximum value of $\mu$ that can be used, while ensuring that envelope detection remains possible at the receivers?

(b) [10 points] Assume that the value of $\mu$ found above is used. We also note that AM transmitters put maximum value constraints on two parameters: the average transmitted power $S_T$, and the peak transmitted power $A_{max}^2$, where $A_{max}$ is the maximum absolute value the modulated signal can take. If both the constraints are to be exactly satisfied simultaneously, derive a relation between $A_{max}$ and $S_T$.

(c) [10 points] Repeat part (b) if $\mu = \frac{3}{4}$ is used.
Problem 9: [20 points] Define a random process $Y(t) = \cos(2\pi f_c t + \Theta_1) \cdot \cos(2\pi f_c t + \Theta_2)$, where $\Theta_1$ and $\Theta_2$ are independent and identically distributed uniform random variables between 0 and $2\pi$. That is, $\Theta_1 \sim U(0, 2\pi)$, and $\Theta_2 \sim U(0, 2\pi)$. Is $Y(t)$ a WSS process? Justify your answer and show the working. If yes, find its power spectral density $S_Y(f)$?