Midterm Examination-II
(Open Books, Open Notes)
Time Allowed: 100 minutes (Nov. 13, 2005; 1:05pm-2:45pm)

“I certify that I have neither received nor given unpermitted aid on this examination and that I have reported all such incidents observed by me in which unpermitted aid is given.”

Signature __________________________

Name _______________________________ Student ID ____________________________

Problem 1 _____________ [30]
Problem 2 _____________ [40]
Problem 3 _____________ [20]
Problem 4 _____________ [30]
Problem 5 _____________ [30]

_____________________________________

TOTAL _________________ [150]
Problem 1: Analog Baseband Audio Transmission [30 points] Consider an analog audio transmission cable used for music transmission at the baseband, where the bandwidth for music can be considered to be 20 kHz. The cable introduces a signal power loss of \( L = 10 \) from source to destination. We want to evaluate the effect of using \( m \) equal-length repeater sections (including the receiver), where the power loss in each segment is \( L_1 \).

Assume that the noise added to the signal is modeled as additive white gaussian noise with power spectral density \( \frac{N_0}{2} \), where \( N_0 = 1 \times 10^{-8} \text{ W/Hz} \). At the destination, a typical baseband receiver is employed whose front end is a unity passband gain lowpass filter with passband characteristics given by:

\[
H(f) = \begin{cases} 
1 & |f| \leq 20 \text{ kHz} \\
\sqrt{5 - \frac{f}{5000}} & 20 \text{ kHz} \leq |f| \leq 25 \text{ kHz} \\
0 & \text{else}
\end{cases}
\]

No additional gain is provided at the receiver. When there are no repeater sections, the destination signal-to-noise ratio \( (\frac{S}{N})_D \) is 40 dB.

(a) [5 points] What is the transmitted signal power \( S_T \)?

(b) [7 points] Now consider that the transmission uses \( m \) repeater sections (including the receiver), but we intend to keep the same transmit power \( S_T \) and same destination signal-to-noise ratio \( (\frac{S}{N})_D \) as in the case of transmission without repeaters. What relation must exist between \( m \) and \( L \) such that the destination signal-to-noise ratio \( (\frac{S}{N})_D \) remains 40 dB for the same transmit power you found in part (a)?

(c) [6 points] Under the condition of part (b), what is the transmitted signal power \( S_T \)?

(d) [6 points] If \( (\frac{S}{N})_D \) is fixed at 40 dB, find \( S_T \) for each value of \( m \) ranging from 2 to 10.

(e) [6 points] If \( S_T \) is fixed at the value computed in part (a), find \( (\frac{S}{N})_D \) for each value of \( m \) ranging from 2 to 10.
Problem 2: Sampling [40 points] Consider a signal $x(t)$ whose Fourier transform $X(f)$ is shown in the figure below (the frequency axis is in Hz).

![Fourier Transform Graph]

The signal $x(t)$ is sampled using an ideal sampling function at a sampling rate $f_s$, and the resulting sampled signal is passed through a low pass filter whose frequency response is given by $H(f) = \cap(f_{\frac{W}{2}})$. 

(a) [10 points] What is the Nyquist sampling rate for $x(t)$? If we sample $x(t)$ at the Nyquist rate, can it be recovered by passing the sampled signal through the low pass filter $H(f)$? If not, why? And if not, how can we recover $x(t)$ from the sampled signal? Explain in legible writing. Legible and brief answers will get lot more credit than long and unreadable explanations.

For each of the following parts, what is the signal $\tilde{x}(t)$ at the output of the low pass filter, when the sampling rate is:

(b) [10 points] $f_s = W$ samples per second.
(c) [10 points] $f_s = 2W$ samples per second.
(d) [10 points] $f_s = 3W$ samples per second.

Problem 3: Time Division Multiplexing [20 points] Eighteen voice signals, each of bandwidth 3400 Hz, are to be multiplexed together using a TDM scheme. Each signal is sampled using flat-top sampling, where the duration of each sampling pulse is 2 $\mu$s. The multiplexing operation includes provision for synchronization by adding two extra pulses, each of duration 2 $\mu$s.

(a) [10 points] If the sampling rate is 10 kHz, what is the spacing between successive pulses of the multiplexed signal?

(b) [10 points] Repeat part (a) if the sampling is carried out at the Nyquist rate?
Problem 4: Frequency Modulation [30 points] A tone-modulated FM signal with modulation index $\beta = 2.0$ is transmitted through an ideal bandpass filter with unity passband amplitude, midband frequency $f_c$ and bandwidth $k f_m$, where $f_c$ is the carrier frequency and $f_m$ is the frequency of the modulating tone, and $k$ is some constant number. You may use the Bessel function plot from your book, replicated here for convenience.

(a) [6 points] If $k = 3$, plot the spectrum of the output of the ideal bandpass filter.
(b) [7 points] For $k = 3$, what are the minimum and maximum values of the envelope?
(c) [8 points] If $k = 5$, plot the spectrum of the output of the ideal bandpass filter.
(d) [9 points] For $k = 5$, what are the minimum and maximum values of the envelope?
Problem 5: Noise and Noise Equivalent Bandwidth [30 points]

(a) [10 points] White noise with two-sided power spectral density $N_0 = 10^{-6}$ Watts/Hz is input to a unity-gain low-pass filter of bandwidth $W$. i.e., $H(f) = 1$ for $|f| < W$, and is zero, otherwise. What should be the bandwidth of the low pass filter if the output noise power is set to be 30 dBm. Be careful about the definition of dBm.

(b) [10 points] Find the noise equivalent bandwidth of a filter whose time domain response is $\text{rect}(t)$.

(c) [10 points] Find the noise equivalent bandwidth of a filter whose frequency response is given in the following figure.