Bayes Decision Rule:

\[ R(x_i | x) = \sum_{j=1}^{\infty} \lambda_j (x_i | w_j) P(w_j | x) \]

Bayes Risk resulting min. overall risk.

Example: 2-category case:

\[ x_1 \Rightarrow \text{decide on } w_1 \]
\[ x_2 \Rightarrow \text{decide on } w_2 \]

Shorthand:

\[ R(x_1 | x) = \lambda_{11} P(w_1 | x) + \lambda_{12} P(w_2 | x) \]
\[ R(x_2 | x) = \lambda_{21} P(w_1 | x) + \lambda_{22} P(w_2 | x) \]

Decide \( w_1 \) if \( R(x_1 | x) < R(x_2 | x) \)

\[ (\lambda_{21} - \lambda_{11}) P(w_1 | x) > (\lambda_{22} - \lambda_{12}) P(w_2 | x) \]

in most cases \( (\lambda_{21} - \lambda_{11}) & (\lambda_{12} - \lambda_{22}) \) are +ve

\[ (\lambda_{21} - \lambda_{11}) p(x | w_1) P(w_1) > (\lambda_{12} - \lambda_{22}) p(x | w_2) P(w_2) \]
Since $d_{21} > d_{11}$, so

\[
\frac{p(x|w_1)}{p(x|w_2)} > \frac{d_{12} - d_{22}}{d_{21} - d_{11}} \frac{p(w_2)}{p(w_1)}
\]

Likelihood Ratio

Independent of $x$
Example 2:

\[ \text{Action } \alpha_i = w_i \quad \text{Min Error Rate} \]

\[ \lambda(\alpha_i | w_j) = \left\{ \begin{array}{ll}
1 & i = j \\
0 & i \neq j
\end{array} \right. \]

\[ \Rightarrow \text{zero-one loss function.} \]

Then:

\[ R(\alpha_i | x) = \sum_j \lambda(\alpha_i | w_j) P(w_j | x) \]
\[ = \sum_{j \neq i} P(w_j | x) \]

\[ P(w_i | x) = \hat{y} - P(w_i | x) \]

Decide on \( w_i \) if \( P(w_i | x) > P(w_j | x) \) for \( j \neq i \)

\[ \text{label} = \arg \max_i P(w_i | x) \]
Sometimes we do not know the exact prior. I can design a classifier to work with a range of priors, to minimize the maximum overall risk, i.e., worst risk is as low as possible.

**MINIMAX CRITERION**

2 category case: \[ R_1 = \text{decide } w_1 \]
[\[ R_2 = \text{decide } w_2 \]

Recall: overall risk \[ R = \int R(x|\mathbf{x})p(\mathbf{x})d\mathbf{x} \]
and \[ R_i(\mathbf{x}_i|\mathbf{x}) = \sum_j p(\mathbf{w}_j|\mathbf{x}_i, \mathbf{x}) \]
\[ R = \int_{R_1} \left[ \lambda_{11} P(w_1|x) + \lambda_{12} P(w_2|x) \right] p(x) \, dx \\
+ \int_{R_2} \left[ \lambda_{21} P(w_1|x) + \lambda_{22} P(w_2|x) \right] p(x) \, dx \\
= \int_{R_1} \left[ \lambda_{11} p(x|w_1) P(w_1) + \lambda_{12} p(x|w_2) P(w_2) \right] \, dx \\
+ \int_{R_2} \left[ \lambda_{21} p(x|w_1) P(w_1) + \lambda_{22} p(x|w_2) P(w_2) \right] \, dx \\
= 1 - P(w_1) \\
\]

\[ P(w_2) = \frac{\int_{R_2} p(x|w_2) \, dx}{1 - \int_{R_2} p(x|w_1) \, dx} \]

\[ R = \lambda_{22} + (\lambda_{12} - \lambda_{22}) \int_{R_1} p(x|w_2) \, dx \\
+ P(w_1) \left[ \lambda_{11} - \lambda_{22} + (\lambda_{21} - \lambda_{11}) \int_{R_2} p(x|w_2) \, dx \right] \\
- (\lambda_{12} - \lambda_{22}) \int_{R_1} p(x|w_2) \, dx \]

\[ \text{Error} = \begin{cases} 
\text{error at } x = \theta \\
0 
\end{cases} \quad \text{for min-max soln.} \]

\[ R = \lambda_{22} + (\lambda_{12} - \lambda_{22}) \int_{R_1} p(x|w_2) \, dx \\
= \lambda_{11} + (\lambda_{21} - \lambda_{11}) \int_{R_2} p(x|w_1) \, dx \]
Classifiers, Discriminant Functions, & Decision Boundaries -
Multi-category case:

Discriminant functions: \( g_i(x) \), \( i = 1, \ldots, c \)

Assign \( x \) to \( w_j \) if \( g_i(x) > g_j(x) \) \( \forall j \neq i \)

Bayes Classifier:

General case: \( g_i(x) = -R(\xi_i | x) \)

Min.-Error-Rate Case: \( g_i(x) = P(\xi_i | x) \)

For any set of descriptor functions, each another-
the comparisons of them if (which is monotonically increasing)