MATH 230: Probability

Lec # 01

Life is full of uncertainties

- It is likely to rain in the afternoon.
- Perhaps I get promotion Next year.

The word “likely”, “perhaps” and “chance” convey the idea of uncertainty.

Experiment:

An experiment is a process that generates a result. A single performance of an experiment is called a trial. The result of an experiment is called an outcome.

Deterministic experiment Vs Probabilistic experiment:

Deterministic experiment:

An experiment whose outcomes can be predicted with certainty

- A jar contains 100 White balls, one ball is selected, it cannot be Black.
- You know the initial deposit and interest rate then you can determine the amount after year

Here experiment conditions determine the outcomes i.e. you know the all the data.

Probabilistic experiment:

An experiment whose outcomes cannot be predicted with certainty

- A fair coin is tossed its outcome may be Head or Tail.
- You roll a dice you don’t know what will be the outcome, it may be 1, 2, 3, 4, 5 or 6.
- You roll the dice it comes up 6.

In these experiments, the element of chance is involved. These experiments are also known as Random experiments.

Some interesting probability questions:

The Monty Hall Problem:

Suppose on a game show you are shown three doors and told that one door hides a car while others hide goats. You pick a door, say No. 1, and the host, who knows what is behind the doors, opens another door, say No. 3, which has a goat. He then asks you whether or not you want to switch to No. 2. Is it beneficial for you to switch your choice?
Two Envelopes:
You have two indistinguishable envelopes that each contains money. One contains twice as much as the other. You may pick one envelope and keep the money it contains. You pick at random, but before you open the envelope, you are offered the chance to take the other envelope instead. Should you switch or not?

The Birthday Problem:
How many people should be in a group before we can say with more than an even chance (i.e., with at least $1/2$ probability) that at least two people in the group share a birthday?

Anyboys:
In a family with two children, one of the children is a girl. What is the probability that the other is a girl as well?

The Hat Problem:
Suppose 10 people take of their hats and toss them up. As they come down, each person randomly grabs a hat. What is the probability that no person receives his or her own hat?
What if there are $n$ people? What happens as $n \to \infty$?

Sleeping Beauty:
Sleeping Beauty volunteers to undergo the following experiment and is told all of the following details. On Sunday she is put to sleep. A fair coin is then tossed to determine which experimental procedure is undertaken. If the coin comes up heads, Beauty is awakened and interviewed on Monday, and then the experiment ends. If the coin comes up tails, she is awakened and interviewed on Monday, given an amnesia-inducing drug that makes her forget Monday, put to sleep again, and reawakened on Tuesday. In this case, the experiment ends after she is interviewed on Tuesday.
Any time Sleeping Beauty is woken and interviewed, she is asked, "What is your belief now for the proposition that the coin landed heads?"