MATH 230: Probability

Lec # 06

Examples:

Example: An urn contains \( n \) balls; one of them is special ball. If \( k \) balls are chosen at random, what is the probability that special ball is chosen?

Sol:

\[
P(\text{special ball}) = \frac{\binom{1}{1} \times \binom{n-1}{k-1}}{\binom{n}{k}} = \frac{k}{n}
\]

Example: if a die is rolled 4 times what is the probability of getting atone 6?

Sol:

\[
P(\text{at least on 6}) = \frac{\# \text{ of Successful events}}{\text{Total } \# \text{ of events}}
\]

\[
P(\text{at least on 6}) = \frac{\binom{4}{1} \times 5^3 + \binom{4}{2} \times 5^2 + \binom{4}{3} \times 5^1 + \binom{4}{4} \times 5^0}{6^4}
\]

Example: A deck of 52 cards is shuffled what is the probability that 4 top card are

(1) Different denominations?

(2) Different suits?

Sol: Let \( D = \text{Different denominations} \) and \( D' = \text{Different suits} \)

\[
P(D) = \frac{52 \times 48 \times 44 \times 40}{52 \times 51 \times 50 \times 49} = 0.6761
\]

\[
P(D') = \frac{52 \times 39 \times 26 \times 13}{52 \times 51 \times 50 \times 49} = 0.1055
\]

Example: there are 5 hotels in a certain town. If 3 people check into the hotels in a day, what is the probability that they each check in a different hotel?

Sol:

\[
P(\text{diff hotel}) = \frac{5 \times 4 \times 3}{5 \times 5 \times 5} = 0.48
\]

Example: There are \( n \) socks, 3 of which are red, in a drawer. What is the value of \( n \) if, when 2 of the socks are chosen randomly, the probability that they are both red is \( \frac{1}{2} \)?

Sol:

\[
P(R_1 \cap R_2) = \frac{\binom{3}{2}}{\binom{n}{2}} = \frac{1}{2} \Rightarrow n = 4
\]
Proposition:

\[
P(E_1 \cup E_2 \cup \cdots \cup E_i) = \sum_{i=1}^{n} P(E_i) - \sum_{i_1 < i_2} P(E_{i_1} \cap E_{i_2}) + \cdots + (-1)^{r+1} \sum_{i_1 < i_2 < \cdots < i_r} P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_r}) + \cdots + (-1)^{n+1} P(E_1 \cap E_2 \cap \cdots \cap E_n)
\]

Example: A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes. What percentage of males smokes neither cigars nor cigarettes?

Sol: Let \( E \) = male smokes cigarettes and \( F \) = male smokes cigars

\[
P(E) = 0.28, \quad P(F) = 0.07, \quad P(E \cap F) = 0.05
\]

\[
P((E \cup F)^c) = \% \text{ of males that smoke neither cigarettes or cigars}
\]

\[
P((E \cup F)^c) = 1 - P(E \cup F)
\]

\[
P((E \cup F)^c) = 1 - 0.28 - 0.07 + 0.05 = 0.7 \quad \text{or} \quad 70\%
\]

Exercise: Suppose that each of \( N \) men at a party throws his hat into the center of the room. The hats are first mixed up, and then each man randomly selects a hat. What is the probability that none of the men selects his own hat?

Conditional Probability:

Definition:

If \( P(F) > 0 \) then probability of event \( E \) given \( F \) has occurred is defined as

\[
P(E|F) = \frac{P(E \cap F)}{P(F)}
\]
Example: A fair coin is tossed twice find the probability that second flip is $E = \{Head\}$ given first flip is $F = \{Head\}$?

$$S = \{HH, HT, TH, TT\}$$
$$E = \{Head\}, \quad F = \{Head\}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$P(E|F) = \frac{1/4}{2/4} = \frac{1}{2}$$

Example: Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?

Sol: Let $E = \{\text{at least one die is a six}\}$ and $F = \{\text{two die land on different #'s}\}$

$$P(F) = \frac{30}{36} = \frac{5}{6}$$

$$P(E \cap F) = \frac{10}{36} = \frac{5}{18}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{5/6}{5/18} = \frac{6}{18} = \frac{1}{3}$$

Example: A couple has 2 children. What is the probability that both are girls if the older of the two is a girl?

Sol: Let $E = \{\text{both children are girls}\}$ and $F = \{\text{eldest child is a girl}\}$

$$P(F) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(E \cap F) = \frac{1}{4}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/4}{1/2} = \frac{1}{2}$$