Problem 1. Consider rolling a fair die. What is the probability of rolling a 4?

Answer. When we say fair, we mean that all of the outcomes 1, 2, 3, 4, 5, 6 are equally likely. Since probabilities sum to 1, each outcome has probability $1/6$. In this problem, we are asked about the probability of a single outcome so the probability is $1/6$.

Problem 2. Consider rolling a fair die. What is the probability of rolling a 4 or a 6?

Answer. The sample space looks like $S = \{1, 2, 3, 4, 5, 6\}$ which each outcome being equally likely with probability $1/|S| = 1/6$.

The event we are interested in is $E = \{4, 6\}$. The probability of this event (recall equally likely outcomes) is

$$P = \frac{|E|}{|S|} = \frac{2}{6} = \frac{1}{3}$$

Problem 3. You roll a fair die. You do not look at the result. Your friend tells you that you rolled an odd number. What is the probability that you rolled a 6?

Answer. 0

Problem 4. You roll a fair die. You do not look at the result. Your friend tells you that you rolled an odd number. What is the probability that you rolled a 3 or a 5?

Answer. Here the sample space would be $S = \{1, 3, 5\}$ with each outcome being equally likely. The event we are interested in is $E = \{3, 5\}$. Then $P(E) = |E|/|S| = 2/3$.

Problem 5. How many five card hands are there?

Answer. We can think about putting five cards from a standard deck into a sequence. The number of such sequences is

$$52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$$

But we were asked about hands, not sequences. If we interchange any two cards that we are holding, we don’t regard that as a new hand. So, there is an overcounting by the number of arrangements of any five given cards which is 5!

Then the number of hands is

$$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!} = \frac{52!}{5!} = \frac{52!}{5! \cdot 47!} = \binom{52}{5}$$

Problem 6. What is the probability of drawing a five card hand consisting entirely of red cards?

Answer. Consider first how many 5-sequences of red cards there are:

$$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$$

Any rearrangement of such a sequence is considered the same hand, so we have overcounted by 5! and the number of 5 card hands with all cards red is

$$\binom{26}{5}$$
\[
\frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22}{5!} = \frac{26!/21!}{5!} = \frac{26!}{5!21!} = \binom{26}{5}
\]

There are \( \binom{52}{5} \) 5 card hands in total, each equally likely so the number of 5 card hands with all cards red is

\[
\binom{26}{5} / \binom{52}{5}
\]

**Problem 7.**

What is the probability of being fairly dealt a five card hand that consists of the ace of hearts, two of hearts, 3 of clubs, 4 of spades, and five of diamonds?

**Answer.** This describes exactly one unique five card hand and there are \( \binom{52}{5} \) possible hands. So the probability is

\[
\frac{1}{\binom{52}{5}}
\]

**Problem 8.** What is the probability of being dealt a five card hand that contains the ace of hearts, two of hearts, 3 of clubs, and 4 of spades?

**Answer.** Four cards are fixed, we need to choose one of 48 so there are 48 such hands, each equally likely out of \( \binom{52}{5} \) equally likely hands. So the probability is

\[
\frac{48}{\binom{52}{5}}
\]

**Problem 9.** What is the probability of being dealt a five card hand that does not contain the case of hearts?

**Answer.** First consider how many five card hands contain the ace of hearts. There is one way to choose the ace of hearts, there are \( \binom{51}{4} \) ways to choose the remaining four cards.

The probability of a five card hand that does contain the ace of hearts is

\[
\binom{51}{4} / \binom{52}{5}
\]

Then the probability of this event not occurring is

\[
1 - \binom{51}{4} / \binom{52}{5}
\]

**Problem 10.** Consider a standard well shuffled 52 card deck of cards. What is the probability of being dealt a three card hand containing no aces?

**Answer.** There are \( \binom{52}{3} \) possible 3 card hands.

The number of possible 3 card sequences with no aces is 48 \cdot 47 \cdot 46. This overcounts by 3! since order in a hand does not matter, so number of three card hands with no aces is

\[
\frac{48 \cdot 47 \cdot 46}{3!} = \frac{48!}{3!45!} = \binom{48}{3}
\]
Problem 11. You are dealt 5 cards from a standard well-shuffled 52 card deck. What is the probability of a flush (all cards from the same suit).

Answer. We can think of constructing a flush as a process of first selecting one suit of four and then from 13 cards of that suit selecting 5. Then there are \( 4 \cdot \binom{13}{5} \) ways of doing this. There are \( \binom{52}{5} \) 5 card hands, so the probability is

\[
\frac{4 \cdot \binom{13}{5}}{\binom{52}{5}}
\]

The desired answer then is

\[
\frac{\binom{48}{3}}{\binom{52}{3}}
\]