Unification

“It is an algorithm for determining the substitutions needed to make two predicate calculus expressions match”

Examples of substitution

1. \( \forall X (\text{man}(X) \rightarrow \text{mortal}(X)) \)
   substitute ‘hassan’ for \( X \) => mortal (hassan)

2. If \( p(X) \) and \( p(Y) \) are equivalent then substitution of \( X \) with \( Y \) is required
**Skolemization**

Skolemization replaces each existentially quantified variable with a function that returns the appropriate constant as a function of some or all of the other variables in the sentence.

**Example:**

- $\forall X \exists Y \text{ mother } (Y, X)$

- Value of $Y$ depends on $X$.
  Thus $Y$ can be replaced by $f(X)$ (Skolem function)
father (jack)

\textit{man(father (jack)}

could be written as

\textit{man(X).}
Examples:  \( \text{food}(X, a, \text{good}(Y)) \)

**Legal Expressions**  \( \text{food}(\text{fred}, a, \text{good}(Z)) \) \( \text{food}(Z, a, \text{good}(\text{mood}(Z))) \)

**Substitutions**  \( \{\text{fred}/X, Z/Y\} \) \( \{Z/X, \text{mood}(Z)/Y\} \)

Substitutions Are Referred To As **Bindings**
A variable is said to be “BOUND” to the value substituted for it.

Which Replacement Is Not Allowed?

1. Constant replaced by variables.
2. Variable replaced by constant.
Composition:

Composition of unification substitution is explained as follows:

“If $S$ and $S’$ are two substitution sets, then the composition of $S$ and $S’$ (written as $SS’$) is obtained by:

- applying $S$ to the elements of $S’$ and
- adding the result to $S$.”
Examples:

\[ S = \{ \frac{X}{Y}, \frac{W}{Z} \}, \]
\[ S' = \{ \frac{V}{X} \}, \]
\[ S'' = \{ \frac{A}{V}, \frac{f(B)}{W} \}. \]

Can we have a single substitution for the above

\[ S''S'S = \{ \frac{A}{Y}, \frac{f(B)}{Z} \} \]
Suppose \( S = \{X/Y, W/Z\} \)
\( S' = \{V/Z\} \)

SS’ will be

(Step.1) Applying \( s \) to the elements of \( S' \)
\( S = \{X/Y, W/Z\}, \ S' = \{V/X\} \)

(Applying substitution of \( s \) to \( s' \) to yield \( \{v/y\} \))

(Step.2) Add \( S \) and step 1.

\( P = S'S = \{V/Y, W/Z\} \)
\( P = \{V/Y, W/Z\} \)
\( Q = \{A/V, f(B)/W\} \)

\[ QP = \{A/Y, f(B)/Z\} \]
Most General Unifier

- The unifier should be as general as possible
- The substitution for a variable by a constant restricts the generality

\[ p(X) \{\text{fred}/X\} \quad \text{limits it to fred only.} \]

\[ P(X) \{\text{Z}/X\} \quad \text{does not limits.} \]
**Implementation of Unification**

Using “List format”

<table>
<thead>
<tr>
<th><strong>PC Syntax</strong></th>
<th><strong>List Syntax</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><code>p(a,b)</code></td>
<td><code>(p a b)</code></td>
</tr>
<tr>
<td><code>p(f(a), g(X,Y))</code></td>
<td><code>(p(f a)(g X Y))</code></td>
</tr>
<tr>
<td><code>p(x) ^ q(y)</code></td>
<td><code>((p x) ^ (q y))</code></td>
</tr>
</tbody>
</table>
Examples of Unification.

Unify

\(((\text{parents } X(\text{father } X) (\text{mother } \text{bill})),
\text{(parents } \text{bill} (\text{father } \text{bill})y))\).

Complete Substitution

\{\text{bill}/X, \text{mother } (\text{bill})/Y\}
1. unify((parents X (father X) (mother bill)), (parents bill (father bill) Y))

Unify first elements
and apply
substitutions to rest

2. unify(parents, parents)

return {}

3. unify((X (father X) (mother bill)), (bill (father bill) Y))
1. unify((parents X (father X) (mother bill)), (parents bill (father bill) Y))

Unify first elements and apply substitutions to rest

return {}

2. unify(parents, parents)

3. unify((X (father X) (mother bill)), (bill (father bill) Y))

Unify first elements and apply substitutions to rest

return {bill/X}

4. unify(X,bill)

5. unify(((father bill) (mother bill)), ((father bill) Y))
Logic – Based Financial Advisor

Function: To help a user decide whether to invest in a saving A/C or the stock market or both
Policy:

1. Saving inadequate - should invest in saving A/c regardless of income

2. Saving adequate and adequate income - More profitable option of stock investment

3. Lowe Income & adequate savings - Split surplus between saving and stock investment
**Adequacy of Saving/Income**: (Determined by # of Dependents)

Should have $4000 for each dependant in the bank.

**Adequate Income:**

Steady @ $15,000/year + $4000/dependant.

**Define Predicates:**

```
savings_account (adequate)
savings_account (inadequate)
income(adequate). Income (inadequate).
```
Outputs

Investment (savings)
Investment (stocks)
Investment (combination)

Q. How can we represent outputs?
(by implications, conjunctions or distinction)
**Rules**

- **saving – account (inadequate)** ➞ **investment (savings)**
- **saving – account(adequate) ^ income(adequate)** ➞ **investment(stocks)**
- **saving – account (adequate) ^ income (inadequate)** ➞ **investment (combination)**
Total adequate saving = $5000 \times \frac{\text{No. of Dependents.}}{X}

Define a function to find level of adequacy of savings

\text{min savings} (X) = 5000 \times X

Total adequate income = $15,000 + (4000 \times X)

\text{min income}(X) = $15000 + 4000 \times X)
Expressions to determine adequacy level

\[ \forall X \ amount - saved (X) \land \exists Y \ (dependants \ (Y)) \land \ greater \ (x, \ min \ saving (Y)) \]

\[ \rightarrow \ \text{saving} - \text{account (adequate)} \]

\[ \forall X \ amount - saved (X) \land \exists Y \ (dependants \ (Y)) \land \ greater \ (x, \ min \ saving (Y)) \]

\[ \rightarrow \ \text{savings} - \text{account (inadequate)} \]

Current savings

No. of dependants
**Income Adequacy**

\[ \forall X \ earnings(X,\text{steady}) \land \exists Y \ (\text{dependants} \ Y) \land \text{greater} \ (x, \text{min} \ income(Y)) \]

\[ \rightarrow \ income \ (\text{adequate}) \]

\[ \forall X \ earnings \ (X,\text{steady}) \land \exists Y \ (\text{dependants} \ Y) \land \neg \text{greater} \ (x, \text{min} \ saving(Y)) \]

\[ \rightarrow \ income \ (\text{inadequate}) \]

\[ \forall X \ earnings \ (X,\text{unsteady}) \rightarrow \ income \ (\text{inadequate}) \]
Case Study:

No. of Dependents = 3

Earnings = $25,000 & steady

Amount Saved = $22,000
Unify

No. 10 & 11 with premist of 7

earnings (25000, steady) ^ dependants (3)

earnings (X, steady) ^ dependants (Y)

Substitutions

\{25000/X, 3/Y\}
By Substituting & Evaluating

\[ \text{earnings}(25,000, \text{steady}) \land \text{dependants}(3) \land \neg \text{greater}(25,000, 27,000) \]

\[ \Rightarrow \text{income}(\text{inadequate}) \]

Thus Add New Assertion

12. income(inadequate)
Similarly 9. Can be unified with 4 & 5.

Amount – saved(22000) ^ dependants(3)

Unify with 4

Amount-saved(22000) ^ dependent(3) ^ greater(22000,15000)

→ Saving – account(adequate).

Add

13. saving – account(adequate)

Final advice is = investment (combination)
1. savings_account(inadequate) ⇒ investment(savings).
2. savings_account(adequate) ∧ income(adequate) ⇒ investment(stocks).

3. savings_account(adequate) ∧ income(inadequate) ⇒ investment(combination).

4. ∀ X amount_saved(X) ∧ ∃ Y (dependents(Y) ∧
   greater(X, minsavings(Y)) ⇒ savings_account(adequate).

5. ∀ X amount_saved(X) ∧ ∃ Y (dependents(Y) ∧
   ¬ greater(X, minsavings(Y))) ⇒ savings_account(inadequate).

6. ∀ X earnings(X, steady) ∧ ∃ Y (dependents(Y) ∧
   greater(X, minincome(Y))) ⇒ income(adequate).

7. ∀ X earnings(X, steady) ∧ ∃ Y (dependents(Y) ∧
   ¬ greater(X, minincome(Y))) ⇒ income(inadequate).

8. ∀ X earnings(X, unsteady) ⇒ income(inadequate).

9. amount_saved(22000).
10. earnings(25000, steady).
11. dependents(3).