Problem: Prove this rule of exponents:
$(ab)^n = a^n b^n$
for every natural number $n$.

Solution:
Let us call this statement $S(n)$.

Basis:
We have to prove that $S(1)$ is true. This can be clearly seen to be true: $(ab)^1$ is $ab$ and so is $a^1 b^1 = ab$.

Note that for the basis we have to prove for $n = 1$ and not $n = 0$ because we have to prove for natural numbers and 0 is not a natural number; 1 is the first natural number. Considering that this confusion – whether to prove for $n = 0$ or $n = 1$ for the basis – would have been there unnecessarily, we would not be deducting marks for this mistake.

Induction:
We begin by assuming that it is true when $n = k$; that is, that $S(k)$ is true:
$S(k) = (ab)^k = a^k b^k$ . . . . . . . . . . . . . . . . . . (3)

With this assumption, we must prove that $S(k+1)$ is also true:
$S(k+1) = (ab)^{k+1} = a^{k+1} b^{k+1}$ . . . . . . . . . . . . (4)

(When using mathematical induction, the student should always write exactly what is to be proved.)

To produce line (4), we will multiply both sides of the assumption -- line (3) -- by $ab$:
$(ab)^k ab = a^k b^k ab$
$= a^k ab^{k+1}$

since the order of factors does not matter,
$= a^{k+1} b^{k+1}$

This is line (4), which is what we wanted to show.

So, we have shown that if the theorem is true for a specific natural number $k$, then it is also true for its successor, $k + 1$.

This theorem is therefore true for every natural number $n$. 