Convert the following grammar $G$ into GNF

$$
S \rightarrow XA \mid BB
B \rightarrow b \mid SB
X \rightarrow b
A \rightarrow a
$$

1. Rewrite $G$ in Chomsky Normal Form (CNF)
   3. It is already in CNF

2. Re-label the variables

   $S$ with $A_1$
   $X$ with $A_2$
   $A$ with $A_3$
   $B$ with $A_4$

   $A_1 \rightarrow A_2 A_3 \mid A_4 A_4$
   $A_4 \rightarrow b \mid A_1 A_4$
   $A_2 \rightarrow b$
   $A_3 \rightarrow a$

3. Identify all productions which do not conform to any of the types listed below

   $A_i \rightarrow A_j x_k \ j > i$
   $Z_i \rightarrow A_v x_k \ j \leq n$
   $A_i \rightarrow a x_k$

   $x_k \in V^*$ (i.e. $x_k$ is some string of zero or more variables)

   $a \in T$ (i.e. 'a' is a single terminal)

4. $A_4 \rightarrow A_1 A_4 \ ............... \ identified$

5. $A_4 \rightarrow A_1 A_4 \ \mid b$

   To eliminate $A_1$ we’ll use the substitution rule

   Substituting for $A_i \rightarrow A_2 A_3 \mid A_4 A_4$
\[ A_4 \to A_2 \ A_3 \ A_4 \mid A_4 \ A_4 \ A_4 \mid b \]

The above two productions still do not conform to any of the types in step 3.
Substituting for \( A_2 \to b \)
\[ A_4 \to b \ A_3 \ A_4 \mid A_4 \ A_4 \ A_4 \mid b \]

Now we have to remove left recursive production \( A_4 \to A_4 \ A_4 \ A_4 \)
\[ A_4 \to b \ A_3 \ A_4 \mid b \mid b \ A_3 \ A_4 \ Z \mid b \ Z \]
\[ Z \to A_4 \ A_4 \mid A_4 \ A_4 \ Z \]

6. At this stage our grammar now looks like

\[ A_1 \to A_2 A_3 \mid A_4 A_4 \]
\[ A_4 \to b A_3 \ A_4 \mid b \mid b \ A_3 \ A_4 \ Z \mid b \ Z \]
\[ Z \to A_4 \ A_4 \mid A_4 \ A_4 \ Z \]
\[ A_2 \to b \]
\[ A_3 \to a \]

All rules now conform to one of the types in step 3.
**But the grammar is still not in Greibach Normal Form!**

7. All productions for \( A_2, A_3 \text{ and } A_4 \) are in GNF
for \( A_1 \to A_2 \ A_3 \mid A_4 \ A_4 \)

Substitute for \( A_2 \& A_4 \) to convert it to GNF
\[ A_1 \to b A_3 \mid b A_3 \ A_4 \mid b A_4 \mid b A_3 \ A_4 \ Z \mid b Z A_4 \]

for \( Z \to A_4 \ A_4 \mid A_4 \ A_4 \ Z \)

Substitute for \( A_4 \) to convert it to GNF
\[ Z \to b A_3 A_4 A_4 \mid b A_4 \mid b A_3 A_4 Z A_4 \mid b Z A_4 \mid b A_3 A_4 A_4 Z \mid b A_4 Z \mid b A_3 A_4 Z A_4 Z \mid b Z A_4 Z \]

8. Finally the Grammar in GNF is
\[ A_1 \to b A_3 \mid b A_3 A_4 A_4 \mid b A_4 \mid b A_3 A_4 Z A_4 \mid b Z A_4 \]
\[ A_4 \to b A_3 A_4 \mid b \mid b A_3 A_4 \ Z \mid b Z \]
\[ Z \to b A_3 A_4 A_4 \mid b A_4 \mid b A_3 A_4 Z A_4 \mid b Z A_4 \mid b A_3 A_4 A_4 Z \mid b A_4 Z \mid b A_3 A_4 Z A_4 Z \mid b Z A_4 Z \]
\[ A_2 \to b \]
\[ A_3 \to a \]