Lahore University Of Management Sciences  
BSc (Honours) Programme  
MS Computer Science Programme

Roll #: SOLUTION

Course Title: Automata and Complexity Theory  
Course Code: CS 311/MATH 352  
Instructor: Humaira Kamal  
Exam: Midterm  
(mid-term/final/others)

Quarter: Winter  
Academic Year: 2002-2003  
Date: Saturday, January 04, 2003  
Time Allowed: 1200-1315 hours  
Total Marks: 75

DO NOT OPEN THIS EXAM UNTIL TOLD TO DO SO.

The instructions below must be followed strictly. Failure to do so can result in serious grade loss.

⇒ You may not
• talk to anyone once the exam begins.
• leave the examination room and then return.

⇒ Keep your eyes on your own paper.

⇒ Read all questions very carefully before answering them.

Specific instructions:

2. Calculator usage: Not required
3. Write in pen/pencil: Any. Please write clearly, avoid overwriting and crossing.
4. Any other instruction(s): Do not write your name on the answer sheet. Count the total number of sheets in this exam paper (There should be a total of 9 sheets including the cover). Solve all questions on the space provided in this question paper. Best of Luck!
Q1) [ 10 points ] Encircle the correct choice.

(a+b) (a+λ)* b + λ is equivalent to:

(a) (a+b)a*b
(b) (a+b)a*b + λ
(c) (a+b)a*ab + λ
(d) none of these

(b+ab)* (a+λ) is equivalent to:

(a) b*(abb*)* + b*(abb*)*a
(b) b*(ab*)* (a+λ)
(c) b* (abb*)*
(d) none of these

(a+b+c)* a (a+b+c)* b (a+b+c)* c (a+b+c)* is equivalent to:

(a) (b+c)* a (a+c)* b (a+b)* c (a+b+c)*
(b) (a+b+c)* a (b+c)* b (c+b)* c (a+b+c)*
(c) (b+c)* abc (a+b+c)*
(d) none of these

(aa+ab*)* is equivalent to:

(a) (aa+ab)*
(b) (b*aaab*)*
(c) (aa+a+b)*
(d) none of these
Regular expression that denotes the language accepted by the above generalized transition graph is
\[ r = r_1^* r_2 (r_4 + r_3 r_1^* r_2)^* \]
How would the above expression change if the self loop with \( r_1 \) was absent?

Expression: \[ \gamma_2 (\gamma_4 + \gamma_3 \gamma_2)^* \]

Q2) [10 points]

\( \Sigma = \{a, b\} \)

Construct the DFA for the language in which every 'a' is either immediately preceded by or immediately followed by 'b'. Also specify what information each state represents.

Some sample strings in the language:
- baab
- aba
- bbabb

Some sample strings NOT in the language:
- aaa
- aab
Q3) [10 points] Derive the regular expression for the language accepted by the following NFA. For full credit show all the steps clearly.

\[ \Sigma = \{0,1,2\} \]

1) Single final state

2) Remove state 0

3) Remove state 2

\[
\begin{align*}
(2+02) + (1+01)(01)^* (2+02) \\
(0+\lambda) + (1+01)(01)^* (0+\lambda)
\end{align*}
\]

4) Remove state 3

\[
\begin{align*}
\gamma &= (2+02) + (1+01)(01)^* (2+02) \cdot (02 + (1+01)(01)^* (0+\lambda)) \\
&+ (0+\lambda) + (1+01)(01)^* (0+\lambda)
\end{align*}
\]
Q4) [10 points]

\[ \Sigma = \{a, b\} \]

Give the regular expression for all strings in which any b’s that occur are in groups of an odd number at a time.

Some sample strings in the language:
- aabaabbbab
- bbbbabbbaaab
- aaaa
- ababab

Some sample strings NOT in the language:
- abba
- babbbb

\[
(a + b(bb)^* a)^* (b(bb)^* + \lambda)
\]

or

\[
a^* (b(bb)^* a a^*)^* (b(bb)^* + \lambda)
\]
Q5) [12 points]

A) \[ \Sigma = \{a\} \]

Give the regular grammar for the following language

\[ L = \{ w : |w| \text{ mod } 3 \neq |w| \text{ mod } 2 \} \]

\[
S \rightarrow aaA \mid aaaaA \mid aaaaA \mid aaaaA \\
A \rightarrow aaaaA \mid \lambda
\]

B) \[ [6] \]

What language does the following grammar generate? (Give your answer in set theoretic form i.e. \( L = \{ \ldots \ldots \} \)). Be precise in your answer.

\[
S \rightarrow aSb \mid A \mid B \\
A \rightarrow aA \mid \lambda \\
B \rightarrow bbC \\
C \rightarrow BC \mid \lambda
\]

Is this grammar a regular grammar? (Yes/No) \( \text{NO} \)

\[
L = \{ a^m b^n : n > m > 0 \} \cup \{ a^p b^q : 0 < q < p+2 \}
\]

or

\[
L = \{ a^i b^j : i \neq j-1, \; i, j \geq 0 \}
\]
Q6) [5 points]

$L_1 = \{a^n b^m : n > m \geq 1 \}$ is a non-regular language

$L_2 = \{a^p b^q : 1 \leq p \leq q \}$ is a non-regular language

$L_3 = L_1 \cup L_2$

What is $L_3$? Give your answer in set theoretic form.

Is $L_3$ regular or nonregular language? Regular

$L_3 = \{a^n b^m : n, m \geq 1 \}$

Q7) [8 points]

Using the pumping lemma show that the following language is nonregular.

Give the proof concisely, all steps carry marks. Vague arguments carry no credit.

$\Sigma = \{a, b\}$

$L = \{w \in \Sigma^* : n_a(w) = n_b(w) + 2 \}$

Proof by Contradiction:

Assume 'L' is a regular language and it is accepted by an $m$-state DFA.

Take $w = a^m b^{m-2} \in L \quad |w| \geq m$

By Pumping Lemma, $w$ can be decomposed as

$w = xyz \quad \text{s.t.} \quad |xy| \leq m \text{ and } |y| \geq 1$

such that $wy = xy^2z$ is also in $L$

The substring $y$ must be composed of entirely $a$'s

Suppose $|y| = k$

$w_i = mass - k + ki \quad b^{m-2}$

By Pumping Lemma, $m-k+ki$ should be equal to $(m-2)+2$ for all $i \geq 0$

This contradicts the pumping lemma, $m-k+ki = m$ however, $k(i-1) = 0$ for all $i > 1$

Taking $1 \leq k \leq m$
Q8) [10 points] Minimize the following DFA.

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Q8) [10 points] Minimize the following DFA.
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**Diagram:**

- States: 0, 1, 2, 3, 4, 6
- Transitions:
  - 0 → 1 on 'a'
  - 0 → 4 on 'b'
  - 1 → 2 on 'a'
  - 2 → 3 on 'a'
  - 2 → 1 on 'b'
  - 3 → 6 on 'a'
  - 3 → b
  - 6 → 2 on 'a'
  - 6 → b

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**Minimized DFA:**

- States: 0, 1, 2, 3, 6
- Transitions:
  - 0 → 1 on 'a'
  - 0 → 4 on 'b'
  - 1 → 2 on 'a'
  - 2 → 3 on 'a'
  - 2 → 1 on 'b'
  - 3 → 6 on 'a'
  - 3 → b
  - 6 → 2 on 'a'
  - 6 → b