Problem 1 [3 points]
If for an NFA $N = (Q, \Sigma, \delta_N, q_0, F)$ we have at most one choice of state for any state and input symbol (i.e. $|\delta_N(q, a)| = 1$, $\forall q \in Q$ and $\forall a \in \Sigma$), prove that if a DFA $D$ is constructed by using subset construction on $N$ then $D$ would have all the transitions of $N$ plus some additional transitions to a new Dead/Crash state (say $\Phi$) wherever $N$ is missing a transition for a given state and input symbol (refer to the box on p.67 of your textbook).

Problem 2 [5 points]
In order to prove a language to be regular you are required to give an automaton, regular expression. Why can’t we just use pumping lemma for regular languages to prove them regular?

Problem 3 [5+7+11 = 23 points]
Following languages may or may not be regular. In case you think they are regular, just give a basic idea how you are going to construct an FA or a regular expression for them. Otherwise use pumping lemma for regular languages to show that these are non-regular languages.

1. $L = \{xyz \mid x, y, z \in \{0,1\}^* \land |x| = |z| \text{ and number of zeros in } x \text{ and } z \text{ are equal}\}$
2. $L = \{x2x \mid \text{ where } x \in \{0,1\}^*\}$
3. $L = \{a^n \mid n \text{ is not a prime}\}$

Problem 4 [7 points]
Suppose you are given an automaton $A$ which generates the language $L$, that is $L(A) = L$. Now let us perform the following transformations on $A$ to obtain a new automaton $A'$.

1- Change all the accepting states into non-accepting states.
2- Change all the non-accepting states into accepting states.

Prove that, regardless of the fact that $A$ is deterministic or non-deterministic, $L(A') = L^C$. (The argument required here would be similar to one used in Homework 01, problem 02).
Problem 5 [7+11+11 = 29 points]
Give regular expressions for each of the given languages. You may make use of any of
the closure properties of regular languages. To get full credit you must show all
workings and also argue informally (and briefly) why the transformations give you the
correct solution (you may want to go through sections 4.2.1 and 4.2.2 of your textbook).

1. Give $L^c$ where $L = \{w \mid w$ does not contain the substring 011}$
2. Give $L^r$ and $L^c$ when $L = \{w \mid w$ does not begin and end with
the same letter, and contains odd number of 1s}$
3. Give $L^r$ when $L^c = \{w \mid w$ contains even 0s and odd 1s}$

Problem 6 [7+11 points]
You can use the closure properties of regular languages to prove any language $L$ to be
non-regular provided an already known non-regular language $L'$. Start by assuming that
$L$ is regular, and then transform $L$ to $L'$ by applying closure properties for regular
languages. The resulting language must be regular if the closure properties hold, but this
is a contradiction, so $L$ must be non-regular.
Use this technique to prove the following to be non-regular languages, use $L$ to be equal
to: $L = \{0^n\,1^n \mid n \geq 0\}$
1. $L = \{0^n1^n2^{n-m} \mid n \geq m \geq 0\}$
2. $L = \{0^n1^n \mid$ where $n$ and $m$ are co-prime}$

Problem 7 [7+11 points]
Prove or disprove the following statements:

a) “If $L$ is a non-regular language then its complement $L^c$ is also a non-regular
language”.

b) “If $L_1$ and $L_2$ are two non-regular languages then $L_1 \cup L_2$ is also non-regular”. (Hint: solve part a first)