Problem No. 1 (40 points)
Give regular expressions recognizing the following languages. In all cases, alphabet is \{0, 1\}.

i) \{w | w begins with a 1 and ends with a 0 or w begins with a 0 and ends with a 1\}

\[1(0+1)*0+0(0+1)*1\]

ii) \{w | w contains at least three 1’s\}

\[(0+1)*1(0+1)*1(0+1)*1(0+1)*\]

iii) \{w | w starts with 0 and has odd length, or starts with 1 and has even length\}

\[0(00+01+10+11)*+(10+11)(00+11+01+10)*\]
iv) \( \{w \mid w \text{ doesn’t contain the substring 110} \} \)

\[(0+(10)*)*1^*\]

v) \( \{w \mid \text{length of } w \text{ is at most 5} \} \)

\[\varepsilon+(0+1)+(0+1)(0+1)+(0+1)(0+1)(0+1)+(0+1)(0+1)(0+1)+(0+1)(0+1)(0+1)\]

vi) \( \{w \mid w \text{ contains at least two 0’s and at most one 1} \} \)

\[00^*00^*(\varepsilon+1)+00^*(\varepsilon+1)00^*+(\varepsilon+1)00^*00^*\]

vii) \( \{w \mid \text{every odd position of } w \text{ is 1} \} \)

\[1(01+11)^* (\varepsilon+0+1)\]

viii) \( \{w \mid w \text{ is any string except 11 and 111} \} \)

\[\varepsilon+((0+1)^*0(0+1)^*+(111(0+1)(0+1)^*)^*)+1\]

ix) \( \{w \mid w \text{ contains a 1 at the 3rd position from end of the string} \} \)

\[(0+1)^*1(0+1)(0+1)\]

x) \( \{w \mid w \text{ neither contain two consecutive 0’s nor two consecutive 1’s} \} \)

\[(\varepsilon+1)(01)^*+(\varepsilon+0)(10)^*\]
Problem No. 2 (20 points)
Consider the following two languages over the alphabet \{0, 1\}:
L_1 = \{w \mid w \text{ contains odd number of 0's}\},
L_2 = \{w \mid w \text{ neither contains two consecutive 0's nor two consecutive 1's}\},
Construct DFA's for both of these languages. Then find another DFA using these two DFA's, which represents L_1 \cap L_2.
Problem No. 3 (20 points)
Consider the following language over alphabet \{a, b\}:
\[ L_3 = \{a^n b^m \mid n, m \geq 0 \text{ and } n \text{ and } m \text{ are multiple of four} \} \]
Is this language regular? If yes, then construct a DFA, else prove intuitively.
(Proof should contain only sensible arguments / intuitions, if you are proving it non-regular)

Food for thought: \{a^n b^n \mid n \geq 0 \} is not a regular language!!!
Problem No. 4 (20 points)
Consider the following language over alphabet \{0, 1, 2, ..., 9\}:
\[ L_4 = \{ w \mid w \text{ contains all integers which are divisible by three} \} \]
Is this language regular? If yes, then construct a DFA, else prove intuitively.
(Proof should contain only sensible arguments / intuitions, if you are proving it non-regular)

Yes, this language is regular, because following DFA exists for it:

\[
\begin{array}{c}
\text{0, 3, 6, 9} \\
\text{0, 3, 6, 9} \\
\text{0, 3, 6, 9} \\
\text{0, 3, 6, 9} \\
\end{array}
\]