Lahore University Of Management Sciences
School of Arts and Sciences

Roll #: Solution

Course Title: Automata Theory
Course Code: CS 311 / MA 352
Instructor: Shahab Baqai
Exam: Midterm

Total Pages: 11 (including this page)
Quarter: Winter
Academic Year: 2006-2007
Date: January 15, 2007
Time Allowed: 100 minutes
Total Marks: 100

The instructions below must be followed strictly. Failure to do so can result in serious grade loss.

DO NOT OPEN THIS EXAM UNTIL TOLD TO DO SO.

⇒ Read all questions very carefully before answering.
⇒ Check the number of papers in the question sheet and make sure the paper is complete.
⇒ Keep your eyes on your own paper.
⇒ You may not
  • Talk to anyone once the exam begins.
  • Leave the examination room and then return.
  • Use any communication device

Specific instructions:

Open book/notes, help sheet: Closed Book & Notes
Calculator usage: None
Write in pen/pencil: Optional
Any other instruction(s): no communication devices, e.g. cell phones etc.

Score

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Q # 1 (10 points)
Consider the NFA, \( M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_0, \{q_4\}) \)

Note: \( \lambda \) represents the empty string.

Construct an equivalent DFA. Show all steps.

Let the DFA \( M' = (Q_D, \{a, b\}, \delta_D, q_D, F_D) \) be equivalent to the NFA \( M \).

Then \( Q_D = \{S' : S' = EClose(S) \text{ where } S \subseteq Q\} \)

\( q_D = Eclose(q_0) \)

\( F_D = \{S \in Q_D : S \cap F \neq \phi\} \)

For each set \( S = \{p_1, p_2, \cdots, p_k\} \subseteq Q_D \) and \( \forall a \in \Sigma, \)

\( \delta_D(S, a) = \bigcup_{i=1}^{m} EClose(r_i) \text{ where } \{r_1, r_2, \cdots, r_m\} = \bigcup_{i=1}^{k} \delta(p_i, a) \)
Q # 2 (10 points)
Consider the DFA \( A = ( \{ p, q, r \}, \{ a, b \}, \delta, p, \{ q \} ) \)

Construct a regular expression for the language accepted by the DFA using state-elimination method

Hence, regular expression of this DFA is: \( a^*b(a+ba*ba*b)^* \)
Q # 3 (10 points)

Design a DFA M that accepts the language $L(M) = \{ w \in \{a, b\}^* : w \text{ does not contain 3 consecutive } b\text{'s} \}$
Q #4 (22 points)
(a) In the proof of the pumping lemma justify
   i. $|xy| \leq m$ (07 points)

   ➢ $m$ represents the number of states in DFA of a regular language. If no state is repeated, then we may read an input of maximum $m$ length. However, if some state is repeated, then length of the accepted string will exceed $m$.
   ➢ Minimum length of the picked string is $m$. So, combined length of $|xy|$ must be at most equal to the minimum total length of the string. We don’t pose any restriction on the length of $z$.

   ii. $|y| \geq 1$ (03 points)

$y$ represents the repeatable portion of the string. If it’s less than one, then all pumped strings will be essentially the same, and we won’t be able to conclude anything about acceptance or rejection of a string.
Using the pumping lemma show that the following language is not regular (12 points)

\[ L(M) = \{ w = v\bar{v} : v \in \{a, b\}^* \text{ and } \bar{v} = \text{ all } a's \text{ in } v \text{ replaced by } b's \text{ and vice versa} \} \]

Assume for contradiction that \( L \) is a regular language. Since \( L \) is infinite, we can apply pumping lemma. Let \( m \) be an integer in pumping lemma, pick a string \( w \) such that \( w \in L, |w| \geq m \)

We pick \( w = a^m b^m \)

We write \( w \) as: 
\[
\begin{align*}
    a^m b^m &= xy z \\
    |xy| &\leq m \text{ and } |y| > 0
\end{align*}
\]

Such that: \( |xy| \leq m \) and \( |y| > 0 \)

Thus:
\[
    xyz = a^m b^m = \underbrace{a...a}_{x} \underbrace{a...a}_{y} \underbrace{ab...b}_{z}
\]

Thus: 
\[
    y = a^k, \quad k \geq 1
\]

For pumping lemma:
\[
    x y^i z \in L \quad i = 0, 1, 2, ...
\]

Thus:
\[
    x y^2 z \in L
\]

\[
    xy^2 z = a^{(m+k)} b^m
\]

So, all a’s in \( v \) are no being replaced by b’s in \( \bar{v} \).

Hence, \( xy^2 z \notin L \).

CONTRADICTION!!

Hence, \( L \) is not regular.
Q # 5 (24 points)

(a) Specify the CFG, \( G \), for the language \( L(G) = \{a^i b^j c^k : i = j \text{ or } i = k \text{ or } j = k \} \)

\[
S \rightarrow A \mid B \mid C \\
A \rightarrow XY \\
X \rightarrow aXb \mid \text{Epsilon} \\
Y \rightarrow cY \mid \text{Epsilon} \\
B \rightarrow aBc \mid Z \\
Z \rightarrow bZ \mid \text{Epsilon} \\
C \rightarrow PQ \\
P \rightarrow aP \mid \text{Epsilon} \\
Q \rightarrow bQc \mid \text{Epsilon}
\]

(b) Construct a NPDA for \( L(G) = \{a^i b^j c^k : i = j \text{ or } i = k \text{ or } j = k \} \)
Q # 6 (24 points)
State whether True or False – To get any credit justify your answer.

(a) \( L_4 = L_1 \cap L_2 \cap L_3 \), where \( L_1 \) and \( L_2 \) are regular and \( L_3 \) is CFL. It is possible that \( L_4 \) will be a regular language.
   True.
   \( L_1 = \{ \text{Epsilon} \} \), \( L_2 = a^* \), \( L_3 = anbn \mid n \geq 0 \), \( L_4 = \text{Epsilon} \)

(b) Let \( L_4 = L_1 L_2 L_3 \)
   If \( L_1 \) and \( L_2 \) are regular and \( L_3 \) is not regular, it is possible that \( L_4 \) is regular.
   True.
   \( L_1 = \{ \text{Epsilon} \} \), \( L_2 = a^* \), \( L_3 = ap \), \( L_4 = a^* \)

(c) \( L_2 = \text{Complement of } L_1 \), where \( L_1 \) is a CFL. It is possible that \( L_2 \) will be a regular language.
   True.
   \( L_1 = (a+b)^+ \), Complement of \( L_1 \) is \( \{ \text{Epsilon} \} \).

(d) \( L_4 = L_1 U L_2 U L_3 \), where \( L_1 \), \( L_2 \), \( L_3 \) are regular languages. \( L_4 \) will always be a context free language.
   True.
   Regular languages are closed under union and all regular languages also context free.

(e) \( L_k = \{ a^p : p \text{ is any prime number less than a very large given integer } k \} \), \( L_k \) is a regular language.
   True.
   Given integer \( k \) means a finite integer, and all prime numbers less than \( k \) will also be finite. So, it will be a regular language.

(f) There is a regular language \( L \) for which there is exactly one regular expression \( R \) with \( L(R) = L \).
   False.
   For every possible regular expression, there can be another expression obtained by concatenating epsilon. So, no language can have a single regular expression.

(g) Intersection of two non-regular languages is always non-regular.
   False.
   \( L_1 = anbn \), \( L_2 = bnan \), Intersection of \( L_1 \) and \( L_2 \) is \( \{ \} \), which is regular.

(h) Given a non-regular language \( L_1 \), \( \{ L_1 U (L_1)^R \} \) – where \( (L_1)^R \) is the reversal of \( L_1 \), will always be a regular language.
   False.
   \( L_1 = a^n b^n \), \( L_2 = b^n a^n \), Union of \( L_1 \) and \( L_2 \) is non-regular.