Computer Exercises Set 1
Due Date: Thursday 26th December 2002

These exercises are to be attempted using MATLAB. You should submit code of each problem separately, as well as a document containing your short observations on each exercise. Submissions should be done electronically at J:\cs533\compEx1\ with your roll number as the folder name.

Useful MATLAB functions that you may need to use:
- RANDN Normally distributed random numbers.
- RAND Uniformly distributed random numbers.
- HIST Histogram.
- PLOT Linear plot.
- MESH 3-D mesh surface.
- CONTOUR Contour plot.
- SURF 3-D colored surface.
- IMAGESC Scale data and display as image.
- MEAN Average or mean value.
- COV Covariance matrix.
- VAR Variance.
- STD Standard deviation.
- EIG Eigenvalues and eigenvectors.

Generating random samples from Normal and Uniform distributions:
Look up randn, rand and hist functions in MATLAB to attempt these exercises.

1. Write a function to generate \( n \) random numbers from a univariate normal distribution of a specified mean and variance. Use this function to generate \( 10^k \) random samples from \( N(10, 4) \) for \( k = 1 \ldots 6 \). Plot the samples in a figure to see what they look like. Plot the histogram of the data in each case, using a reasonable bin size. Comment.

2. Write a function to visualize the univariate Normal Distribution Function in one dimension for a specified mean and variance. Use this function to plot \( N(10, 4) \). Compare with the histograms that you have generated.

   Something to think about: Can you write code to overlay the analytical plot of Ex2 over the histogram of Ex1 to see how well the histogram of random samples matches with the actual curve.

3. Write a function to generate \( n \) random numbers from a multivariate normal distribution of \( d \)-dimensions of a specified mean vector and a diagonal covariance matrix (i.e. the dimensions are independent). The function should return a matrix
of random samples of size $n$-by-$d$. Using this function, generate $10^k$ random samples from

$$
\mu = \begin{bmatrix} 0 \\ 2.5 \end{bmatrix}, \Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}
$$

for $k = 1 \ldots 6$. Plot the samples in a figure to see what they look like. Compute the sample covariance matrix in each case using the “cov” function. Comment on the difference between the sample covariance matrix and the actual covariance matrix. Do the samples seem to be correlated?

*Something to think about: How can the histogram be plotted for this set of samples? Can you write a function to do that, equivalent to the hist function in 1D.*

4. Write a function to visualize the 2D multivariate Normal Distribution with arbitrary mean and covariance matrix (*not restricted to the diagonal case*). Use this function to visualize the following cases of the normal distribution:
   a. Diagonal covariance matrix with $\sigma_1 > \sigma_2$
   b. Diagonal covariance matrix with $\sigma_1 < \sigma_2$
   c. Non-diagonal covariance matrix, with $\rho_{12} = \pm 0.2, \pm 0.5, \pm 0.8$

   You may try different visualization options for 2D functions, like mesh, surf, contour, imagesc.

5. Write a function similar to that in Ex3, but with the restriction on diagonal covariance removed. This means that the function should be able to generate $n$ random numbers from a multivariate normal distribution of $d$-dimensions of a specified mean vector and any arbitrary covariance matrix. Use this function to generate 100 samples from

$$
\mu = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \Sigma = \begin{bmatrix} 4 & 4 \\ 4 & 9 \end{bmatrix}
$$

Plot the generated samples. Compute the sample mean and the sample covariance matrix. Repeat these steps 10 times. Comment on the mean and the variance of each component of the sample mean and sample covariance matrix over 10 trials. Comment.

6. Computer Ex 5 from Duda/Hart on CLT (Pg. 80).